

COMMUNICATION SYSTEMS

Course Code :20EC07

Dr. POORNAIAH BILLA

ANGLE MODULATION

UNIT – II: ANGLE MODULATION

Definition, types of angle modulation: Frequency modulation, Phase modulation, single tone frequency modulation, Narrow band FM(NBFM):time and frequency domain representation, Wide band FM(WBFM):time and frequency domain representation, Transmission bandwidth of FM , Generation of FM : direct method, indirect method, Detection of FM waves: Frequency discrimination method, Phase discrimination method.

Angle Modulation

The other type of modulation in continuous-wave modulation is **Angle Modulation**. Angle Modulation is the process in which the frequency or the phase of the carrier signal varies according to the message signal.

The standard equation of the angle modulated wave is

$$s(t) = A_c \cos \theta_i(t)$$

here,

A_c is the amplitude of the modulated wave, which is the same as the amplitude of the carrier signal, $\theta_i(t)$ is the angle of the modulated wave.

Angle modulation is further divided into frequency modulation and phase modulation. **Frequency Modulation** is the process of varying the frequency of the carrier signal linearly with the message signal.

Phase Modulation is the process of varying the phase of the carrier signal linearly with the message signal.

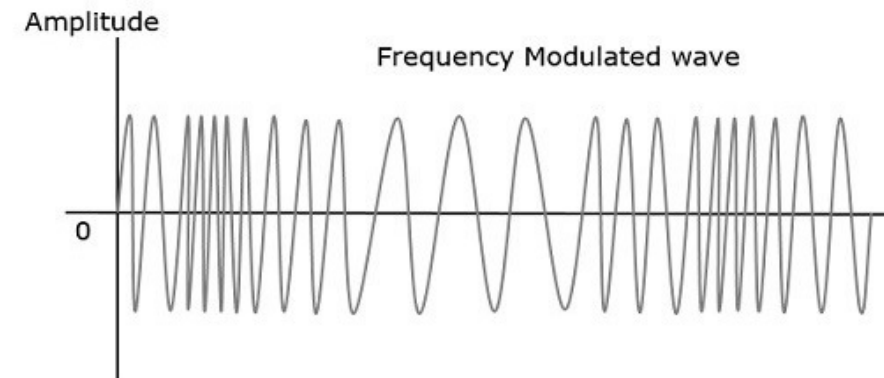
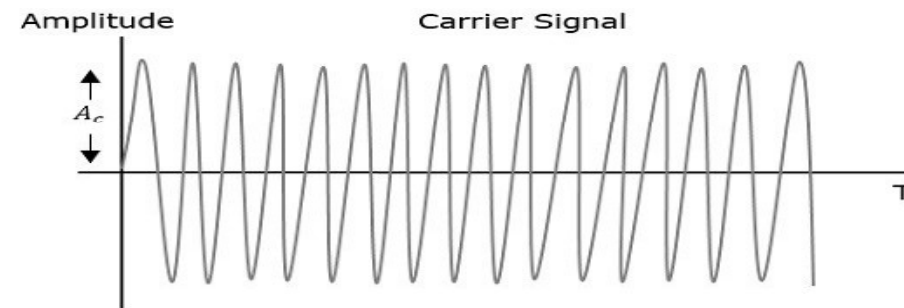
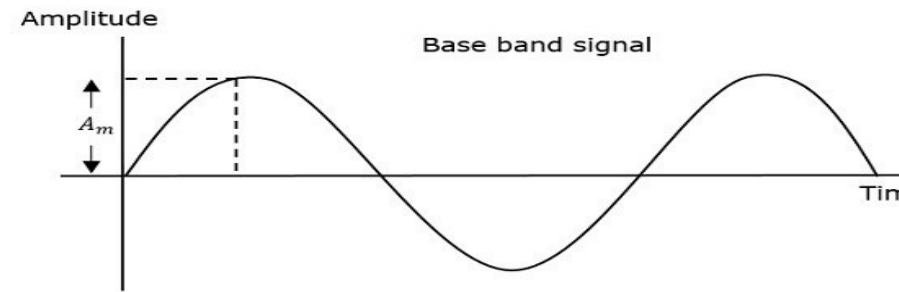
Frequency Modulation

Frequency Modulation

In amplitude modulation, the amplitude of the carrier signal varies. Whereas, in **Frequency Modulation (FM)**, the frequency of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.

Hence, in frequency modulation, the amplitude and the phase of the carrier signal remains constant. This can be better understood by observing the following figures.

The frequency of the modulated wave increases, when the amplitude of the modulating or message signal increases. Similarly, the frequency of the modulated wave decreases, when the amplitude of the modulating signal decreases. Note that, the frequency of the modulated wave remains constant and it is equal to the frequency of the carrier signal, when the amplitude of the modulating signal is zero.



Frequency Modulation

Mathematical Representation:

The equation for instantaneous frequency f_i in FM modulation is

$$f_i = f_c + k_f m(t)$$

Where,

f_c is the carrier frequency

k_f is the frequency sensitivity

$m(t)$ is the message signal

We know the relationship between angular frequency ω_i and angle $\theta_i(t)$ as

$$\omega_i = 2\pi f_i = d\theta_i(t) / dt$$

$$\theta_i(t) = 2\pi \int f_i dt$$

Substitute, f_i value in the above equation

$$\theta_i(t) = 2\pi \int (f_c + k_f m(t)) dt$$

Frequency Modulation

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t) dt$$

Substitute, $\theta_i(t)$ value in the standard equation of angle modulated wave.

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t) dt) \quad \text{This is the **equation of FM wave** .}$$

If the modulating signal is $m(t) = A_m \cos(2\pi f_m t)$ then the equation of FM wave will be

$$s(t) = A_c \cos(2\pi f_c t + (k_f A_m / f_m) \sin(2\pi f_m t))$$

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

Where, $\beta = \text{modulation index} = \Delta f / f_m = k_f A_m / f_m$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by Δf , which is equal to the product of k_f and A_m .

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index.

Frequency Modulation

Narrowband FM:

Following are the features of Narrowband FM.

- This frequency modulation has a small bandwidth when compared to wideband FM.
- The modulation index β is small, i.e., less than 1.
- Its spectrum consists of the carrier, the upper sideband and the lower sideband.
- This is used in mobile communications such as police wireless, ambulances, taxicabs, etc.

Wideband FM:

Following are the features of Wideband FM.

- This frequency modulation has infinite bandwidth.
- The modulation index β is large, i.e., higher than 1.
- Its spectrum consists of a carrier and infinite number of sidebands, which are located around it.
- This is used in entertainment, broadcasting applications such as FM radio, TV, etc.

182 Angle Modulation

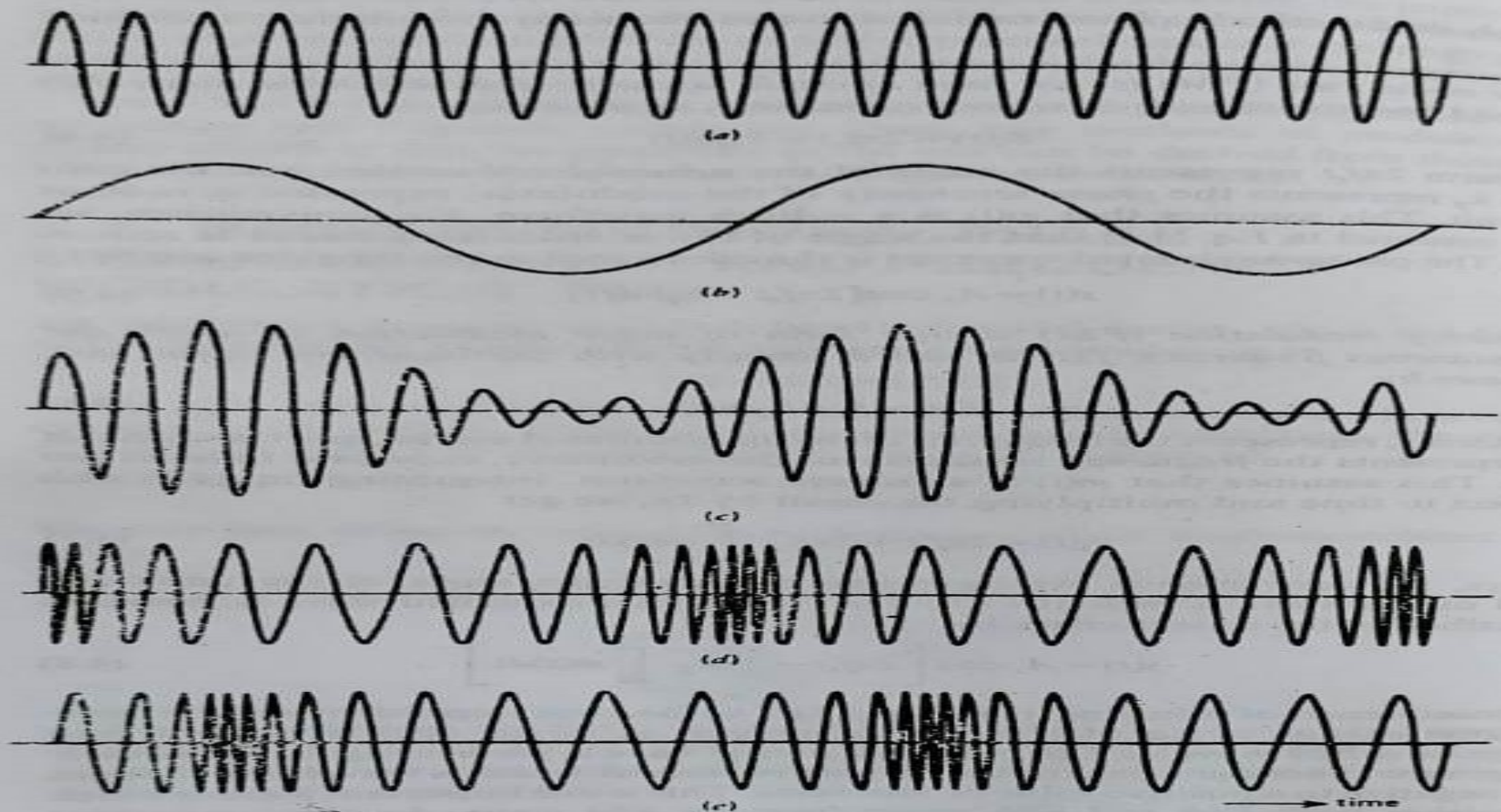


Figure 4.1 Illustrating AM, PM and FM waves produced by a single tone. (a) Carrier wave. (b) Sinusoidal modulating wave. (c) Amplitude-modulated wave. (d) Phase-modulated wave. (e) Frequency-modulated wave.

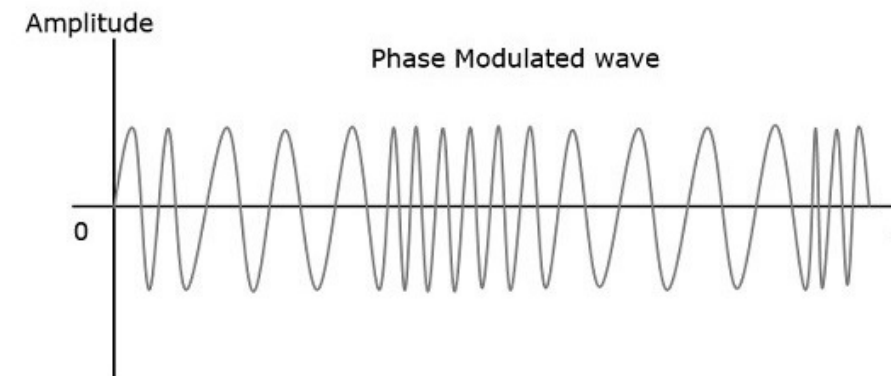
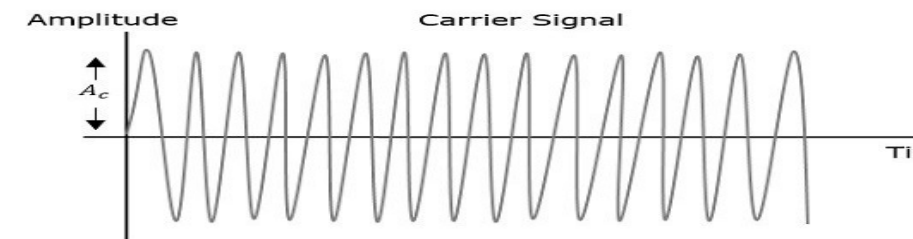
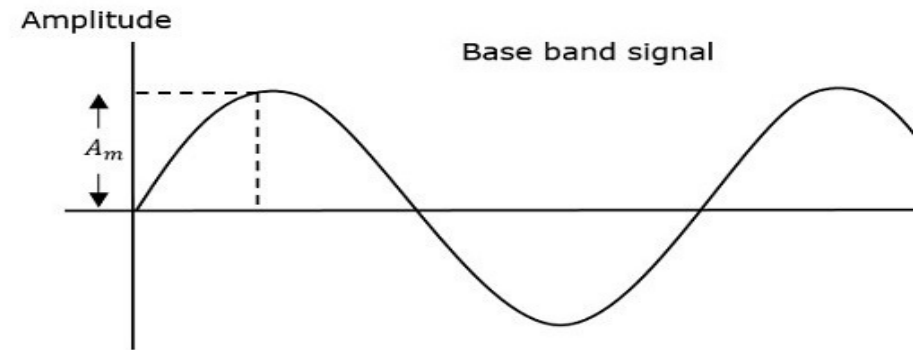
Phase Modulation

Phase Modulation:

In frequency modulation, the frequency of the carrier varies. Whereas, in **Phase Modulation (PM)**, the phase of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.

So, in phase modulation, the amplitude and the frequency of the carrier signal remains constant. This can be better understood by observing the following figures.

The instantaneous amplitude of the modulating signal changes the phase of the carrier signal. When the amplitude is positive, the phase changes in one direction and if the amplitude is negative, the phase changes in the opposite direction.



Phase Modulation

Mathematical Representation:

The equation for instantaneous phase ϕ_i in phase modulation is

$$\phi_i = k_p m(t)$$

Where,

- k_p is the phase sensitivity
- $m(t)$ is the message signal

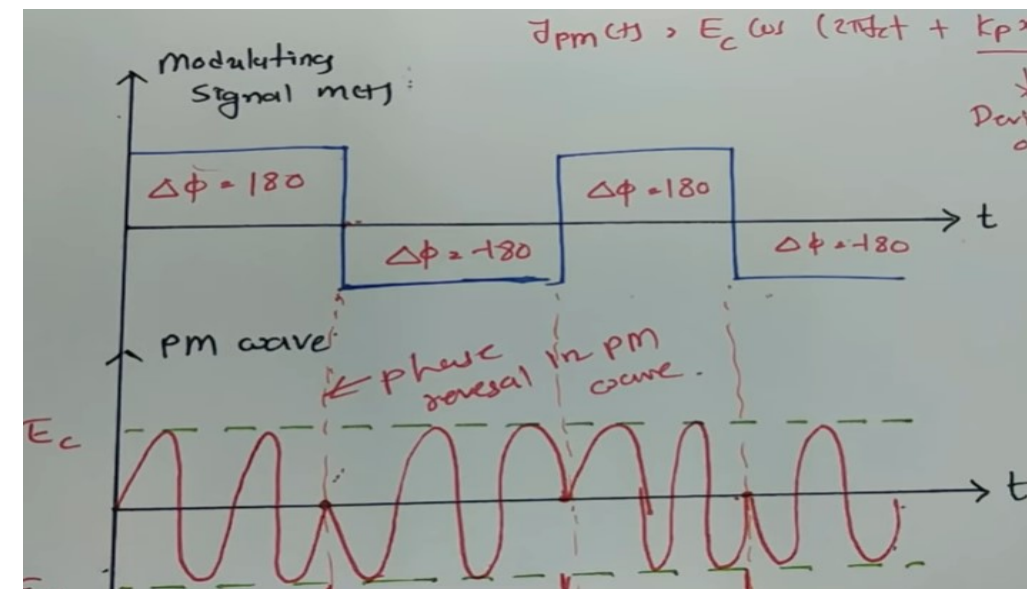
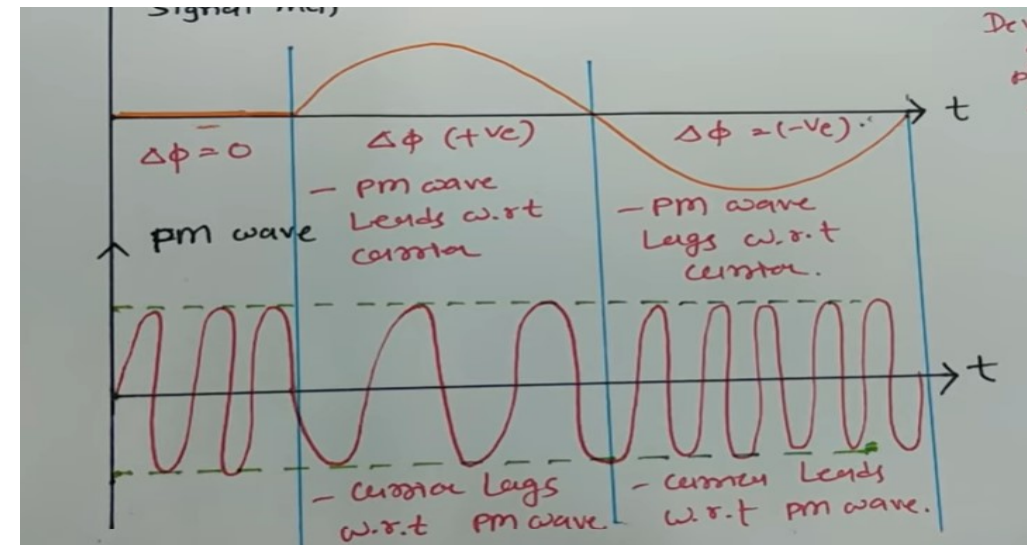
The standard equation of angle modulated wave is

$$s(t) = A_c \cos(2\pi f_c t + \phi_i)$$

Substitute, ϕ_i value in the above equation.

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

This is the **equation of PM wave**.



Phase Modulation

If the modulating signal, $m(t)=A_m\cos(2\pi f_m t)$, then the equation of PM wave will be

$$s(t)=A_c\cos(2\pi f_c t+\beta\cos(2\pi f_m t))$$

Where,

$$\beta = \textit{modulation index} = \Delta\phi = k_p A_m$$

$\Delta\phi$ is phase deviation

Phase modulation is used in mobile communication systems, while frequency modulation is used mainly for FM broadcasting.

Angle Modulation

Comparing Frequency Modulation to Phase Modulation

	FM	PM
1	Frequency deviation is proportional to modulating signal $m(t)$	Phase deviation is proportional to modulating signal $m(t)$
2	Noise immunity is superior to PM (and of course AM)	Noise immunity better than AM but not F
3	Signal-to-noise ratio (SNR) is better than in PM	Signal-to-noise ratio (SNR) is not as good in FM
4	FM is widely used for commercial broadcast radio (88 MHz to 108 MHz)	PM is primarily for some mobile radio services
5	Modulation index is proportional to modulating signal $m(t)$ as well as modulating frequency f_m	Modulation index is proportional to modulating signal $m(t)$

Angle Modulation

Problem 1

A sinusoidal modulating waveform of amplitude 5 V and a frequency of 2 KHz is applied to FM generator, which has a frequency sensitivity of 40 Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

Solution

Given, the amplitude of modulating signal, $A_m=5V$

Frequency of modulating signal, $f_m=2KHz$

Frequency sensitivity, $k_f=40Hz/volt$

We know the formula for Frequency deviation as

$$\Delta f = k_f A_m$$

Substitute k_f and A_m values in the above formula.

$$\Delta f = 40 \times 5 = 200Hz, \Delta f = 40 \times 5 = 200Hz$$

Therefore, **frequency deviation**, Δf is 200Hz

The formula for modulation index is $\beta = \Delta f / f_m$

Substitute Δf and f_m values in the above formula

$$\beta = 200 / 2 \times 1000 = 0.1$$

Here, the value of **modulation index**, β is 0.1 which is less than one. Hence, it is Narrow Band FM.

The formula for Bandwidth of Narrow Band FM is the same as that of AM wave.

$$BW = 2f_m$$

Substitute f_m value in the above formula.

$$BW = 2 \times 2K = 4KHz$$

Therefore, the **bandwidth** of Narrow Band FM wave is 4KHz.

Angle Modulation

Problem 2

An FM wave is given by $s(t)=20\cos(8\pi\times 10^6t + 9\sin(2\pi\times 10^3t))$. Calculate the frequency deviation, bandwidth, and power of FM wave.

Solution

Given, the equation of an FM wave as

$$s(t)=20\cos(8\pi\times 10^6t + 9\sin(2\pi\times 10^3t)).$$

We know the standard equation of an FM wave as

$$s(t)=A_c\cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

We will get the following values by comparing the above two equations.

Amplitude of the carrier signal, $A_c=20V$

Frequency of the carrier signal, $f_c=4\times 10^6\text{Hz}=4\text{MHz}$

Frequency of the message signal, $f_m=1\times 10^3\text{Hz}=1\text{KHz}$

Modulation index, $\beta=9$

Here, the value of modulation index is greater than one.
Hence, it is **Wide Band FM**.

We know the formula for modulation index as $\beta=\Delta f / f_m$

Rearrange the above equation as follows $\Delta f=\beta f_m$

Substitute β and f_m values in the above equation.

$$\Delta f=9 \times 1\text{K}=9\text{KHz}$$

Therefore, **frequency deviation**, Δf is 9KHz.

The formula for Bandwidth of Wide Band FM wave

$$BW=2(\beta+1)f_m$$

Substitute β and f_m values in the above formula.

$$BW=2(9+1)1\text{K}=20\text{KHz}$$

Therefore, the **bandwidth** of Wide Band FM is 20KHz.

Formula for power of FM wave is

$$P_c=A_c^2 / 2R$$

Assume, $R=1\Omega$ and substitute A_c value in the equation.

$$P=(20)^2 / 2(1)=200\text{W}$$

Therefore, the **power** of FM wave is 200 **watts**.

Frequency Modulation

Single tone FM:

The equation for instantaneous frequency f_i in FM modulation is

$$f_i = f_c + k_f m(t)$$

Where,

f_c is the carrier frequency

k_f is the frequency sensitivity

$m(t)$ is the message signal

We know the relationship between angular frequency ω_i and angle $\theta_i(t)$ as

$$\omega_i = 2\pi f_i = d\theta_i(t) / dt$$

$$\theta_i(t) = 2\pi \int f_i dt$$

Substitute, f_i value in the above equation

$$\theta_i(t) = 2\pi \int (f_c + k_f m(t)) dt$$

Frequency Modulation

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t) dt$$

Substitute, $\theta_i(t)$ value in the standard equation of angle modulated wave.

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t) dt) \quad \text{This is the **equation of FM wave** .}$$

If the modulating signal is $m(t) = A_m \cos(2\pi f_m t)$ then the equation of FM wave will be

$$s(t) = A_c \cos(2\pi f_c t + (k_f A_m / f_m) \sin(2\pi f_m t))$$

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

Where, $\beta = \textit{modulation index} = \Delta f / f_m = k_f A_m / f_m$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by Δf , which is equal to the product of k_f and A_m .

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index.

Frequency Modulation

Multitone FM: The modulating signal having more than one frequency components, then the scheme of modulation is known as Multitone FM.

then the equation of FM wave will be

$$s(t) = A_c \cos(2\pi f_c t + \beta_1 \sin(2\pi f_{m1} t) + \beta_2 \sin(2\pi f_{m2} t) + \dots))$$

$$\beta_1 = \text{modulation index} = \Delta f_1 / f_{m1} = k_f A_{m1} / f_{m1}$$

Where, $\beta_2 = \text{modulation index} = \Delta f_2 / f_{m2} = k_f A_{m2} / f_{m2}$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by Δf , which is equal to the product of k_f and A_m .

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index.

Modulation Index β or m_f :- The Modulation index in FM can be defined as the ratio of Frequency deviation (Δf) to the modulating signal frequency f_m .

$$\therefore \beta = \frac{\text{Frequency deviation}}{\text{Modulating Frequency}} = \frac{\Delta f}{f_m}$$

Modulation index (β) can be greater than '1' in FM. It decides the Bandwidths of FM wave also decides no. of sidebands having significant amplitude.

Frequency Deviation (Δf) :- The instantaneous frequency of FM wave varied w.r.t time. The maximum change in instantaneous frequency from the average value carrier frequency ' f_c ' is known as frequency deviation.

$$\Delta f = |k_f m(t)|_{\max} = k_f A_m.$$

In FM, the output of FM swings between two levels because

modulating signal. This swing is called "Carrier swing"

$$\text{Carrier swing} = 2 \Delta f = 2 \times \text{Frequency deviation,}$$

Band width (B.W) :- The FM wave contains infinite no. of sidebands
Thus the B.W of FM signal is "Infinite." But by using Carson's rule
B.W can be defined as

$$\text{B.W} = 2[\Delta f + f_m]$$

Angle Modulation

Relation between FM and PM

The change in phase, changes the frequency of the modulated wave. The frequency of the wave also changes the phase of the wave. Though they are related, their relationship is not linear. Phase modulation is an indirect method of producing FM.

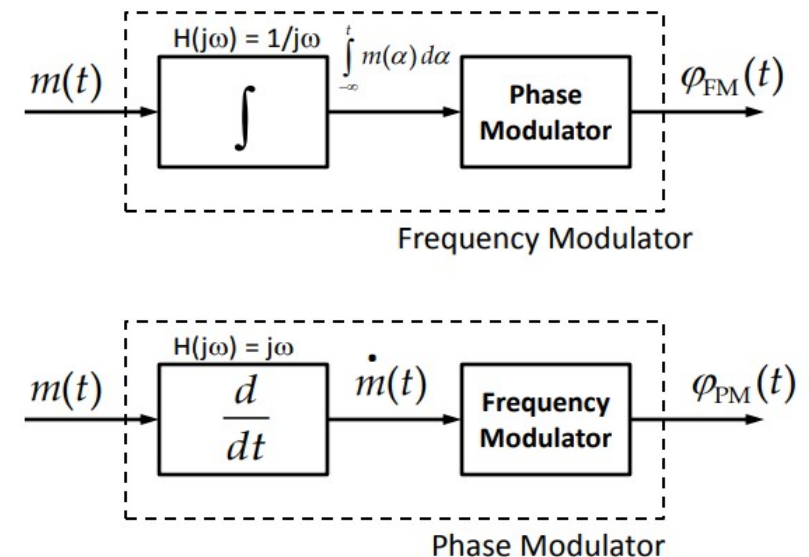
$$\text{FM-} \quad s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t) dt)$$

$$\text{PM-} \quad s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

By comparison FM and PM, FM is generated by integrating the Modulating signal $m(t)$ and applied to PM. PM is generated by differentiating message signal $m(t)$ and applied to Frequency modulator to get PM.

In phase modulation $m(t)$ drives the variation of phase ϕ . In frequency modulation $m(t)$ drives the variation of frequency f .

A Pictorial Way to View the Generation of FM and PM



We require that $H(j\omega)$ be a reversible (or invertible) operation so that $m(t)$ is recoverable.

Frequency Modulation

Carson's Rule for FM bandwidth

The bandwidth of an FM signal is not as straightforward to calculate as that of an AM signal.

A very useful rule of thumb used by many engineers to determine the bandwidth of an FM signal for radio broadcast and radio communications systems is known as Carson's Rule. This rule states that 98% of the signal power is contained within a bandwidth equal to the deviation frequency, plus the modulation frequency doubled. Carson's Rule can be expressed simply as a formula:

$$\text{Total Bandwidth } B_T = 2(1+\beta)f_m = 2(\Delta f + f_m) \qquad \Delta f = \beta f_m$$

FM modulators

FM modulators which generate NBFM and WBFM waves.

Generation of NBFM: When the modulation Index is less than 1 ($\beta \ll 1$), then that FM is called Narrow band FM

We know that the standard equation of FM wave is

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t) dt)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$s(t) = A_c \cos(2\pi f_c t) \cos 2\pi k_f \int m(t) dt - A_c \sin(2\pi f_c t) \sin 2\pi k_f \int m(t) dt$$

For NBFM,

$$2\pi k_f \int m(t) dt \ll 1$$

We know that, $2\pi k_f \int m(t) dt$ is very small,

$$\cos 2\pi k_f \int m(t) dt \approx 1 \quad \sin(2\pi k_f \int m(t) dt) \approx 2\pi k_f \int m(t) dt$$

By using the above relations, we will get the **NBFM equation** as

$$s(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) 2\pi k_f \int m(t) dt$$

$$s(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) \beta \sin(2\pi f_m t)$$

FM modulators

$$= A_c \cos(2\pi f_c t) + \frac{A_c \beta}{2} [\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t]$$

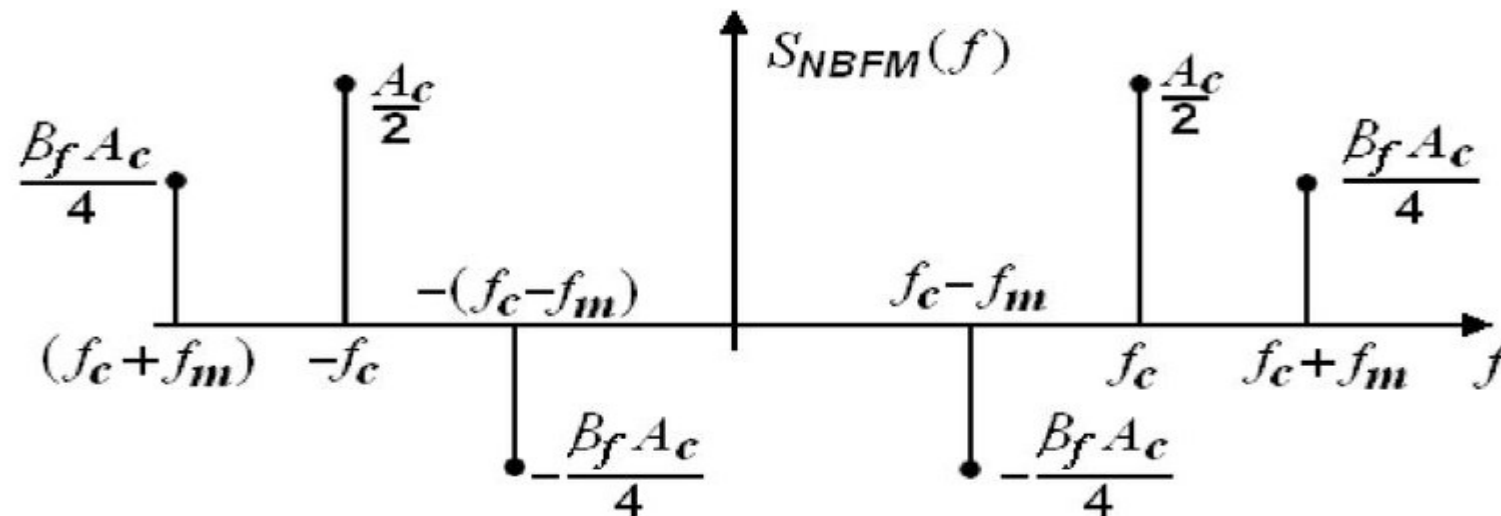
$$S(f) = \frac{A_c}{2} (\delta(f + f_c) - \delta(f - f_c)) + \frac{A_c \beta}{4} (\delta(f + f_c + f_m) + \delta(f - f_c - f_m)) \\ - \frac{A_c \beta}{4} (\delta(f + f_c - f_m) + \delta(f - f_c + f_m))$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

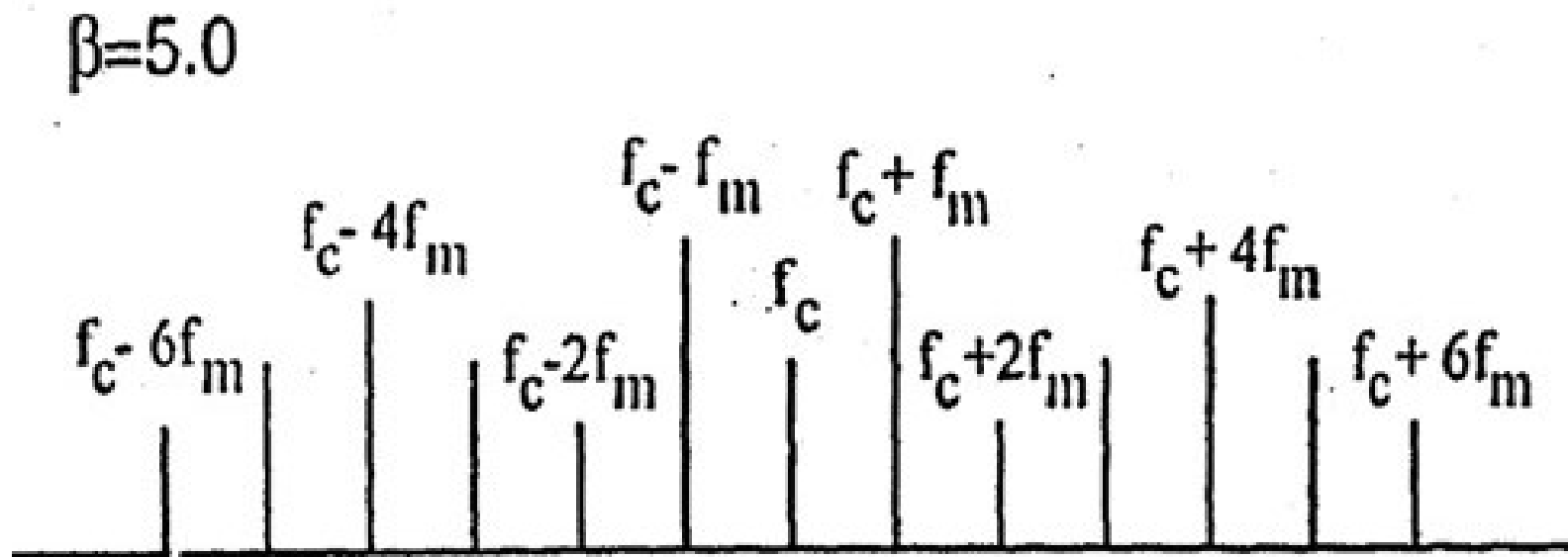
$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$



Spectrum of Frequency Modulation



Narrow Band Frequency Modulation

The block diagram of NBFM modulator is shown in the following figure.

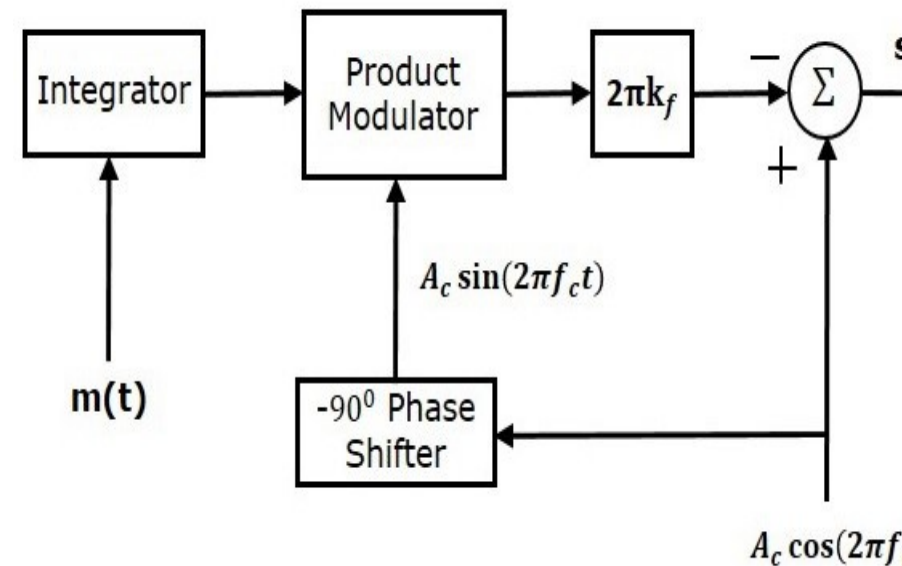
The NBFM is used to **improve the spectrum efficiency**

Here, the integrator is used to integrate the modulating signal $m(t)$. The carrier signal $A_c \cos(2\pi f_c t)$ is the phase shifted by -90° to get $A_c \sin(2\pi f_c t)$ with the help of -90° phase shifter.

The product modulator has two inputs $\int m(t) dt$ and $A_c \sin(2\pi f_c t)$. It produces an output, which is the product of these two inputs.

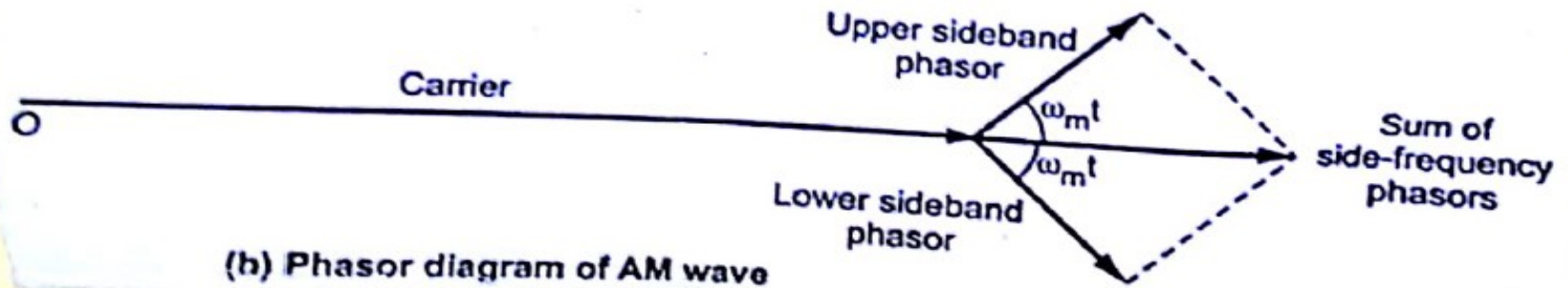
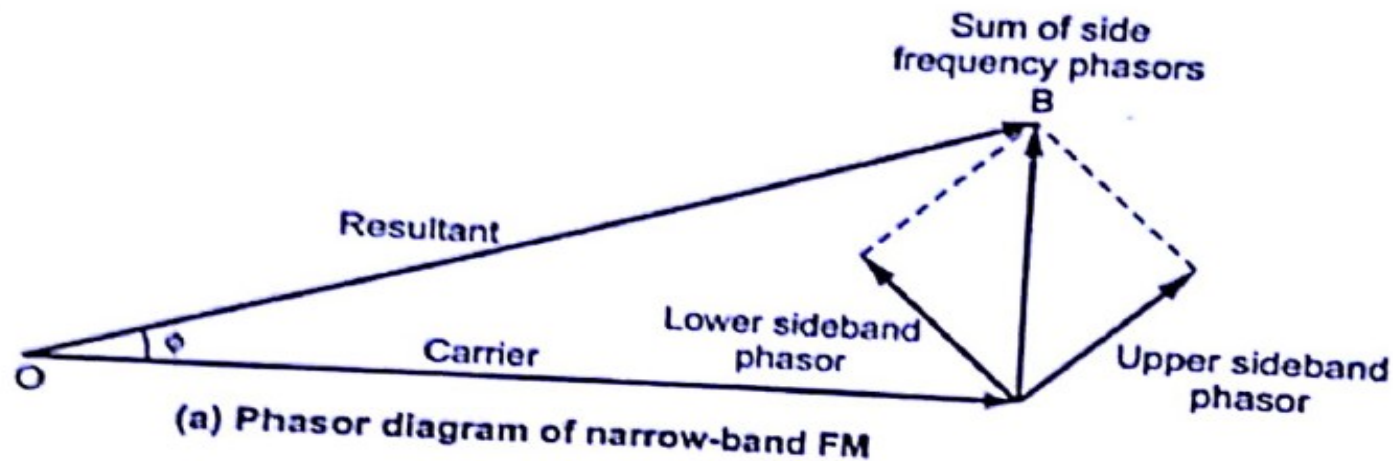
This is further multiplied with $2\pi k_f$ by placing a block $2\pi k_f$ in the forward path. The summer block has two inputs, which are nothing but the two terms of NBFM equation. Positive and negative signs are assigned for the carrier signal and the other term at the input of the summer block. Finally, the summer block produces NBFM wave.

Generate NBFM, the amplitude of carrier is vary due adder circuit, to overcome this problem by using band pass limiters.



Narrow Band Frequency Modulator

The phasor diagram for NBFM is shown below.



Wide Band Frequency Modulator

WBFM:

The modulation index β is large, i.e., higher than 1 then FM modulation is called WBFM. When $\beta \gg 1$, then we can not neglect the terms in FM equation.

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t) dt)$$

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$s(t) = A_c \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))$$

The bandwidth of WBFM is 'Infinite'

Wide Band Frequency Modulator

WBFM: In FM, When the modulation index β is quite large, a large number of side bands are produced and hence the bandwidth of FM is sufficiently large. This type of FM is WBFM.
The time domain representation of WBFM is

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi (f_c + n f_m) t] \quad \text{--- (1)}$$

Where $J_n(\beta)$ is the n^{th} order Bessels function of first kind with argument β

The frequency spectrum of $s(f)$ is obtained by taking Fourier transform of above equation -1

$$s(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - (f_c + n f_m)) + \delta(f + (f_c + n f_m))] \quad \text{--- (2)}$$

Some properties of Bessels function

1. For 'n' is even $J_n(\beta) = J_{-n}(\beta)$

For 'n' is odd $-J_n(\beta) = J_{-n}(\beta)$

i.e. $J_n(\beta) = (-1)^n J_{-n}(\beta)$

2. For small values of β

$$J_0(\beta) \simeq 1$$

$$J_1(\beta) \simeq \frac{\beta}{2}$$

$$J_n(\beta) \simeq 0 \text{ for } n > 1$$

3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

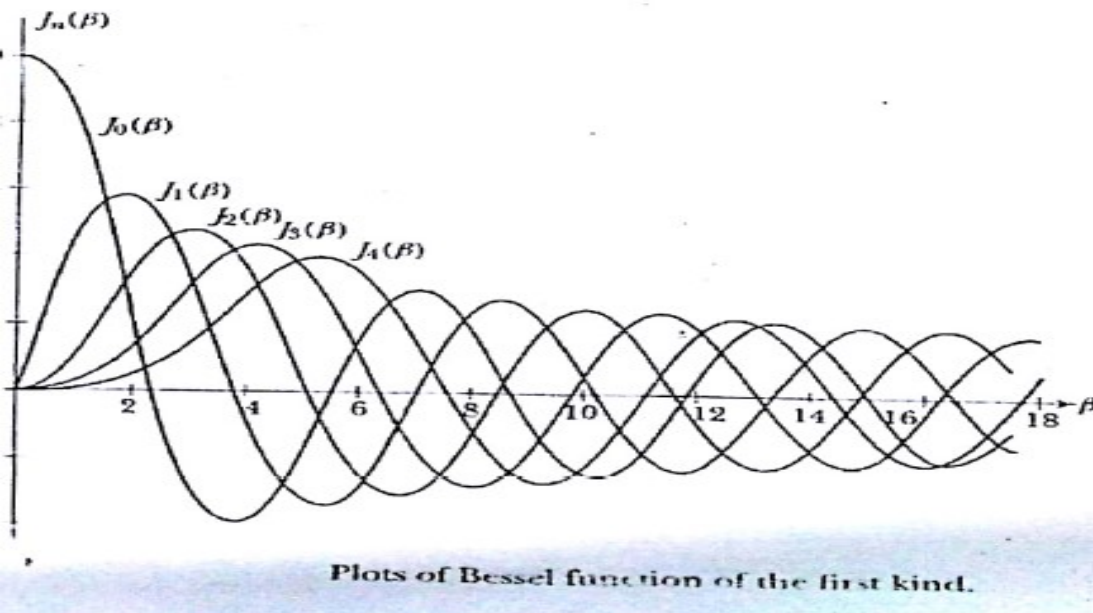
Wide Band Frequency Modulator

WBFM: Thus making use of above properties expanding the equation of WBFM modulated wave for $n=0,1,2,3$

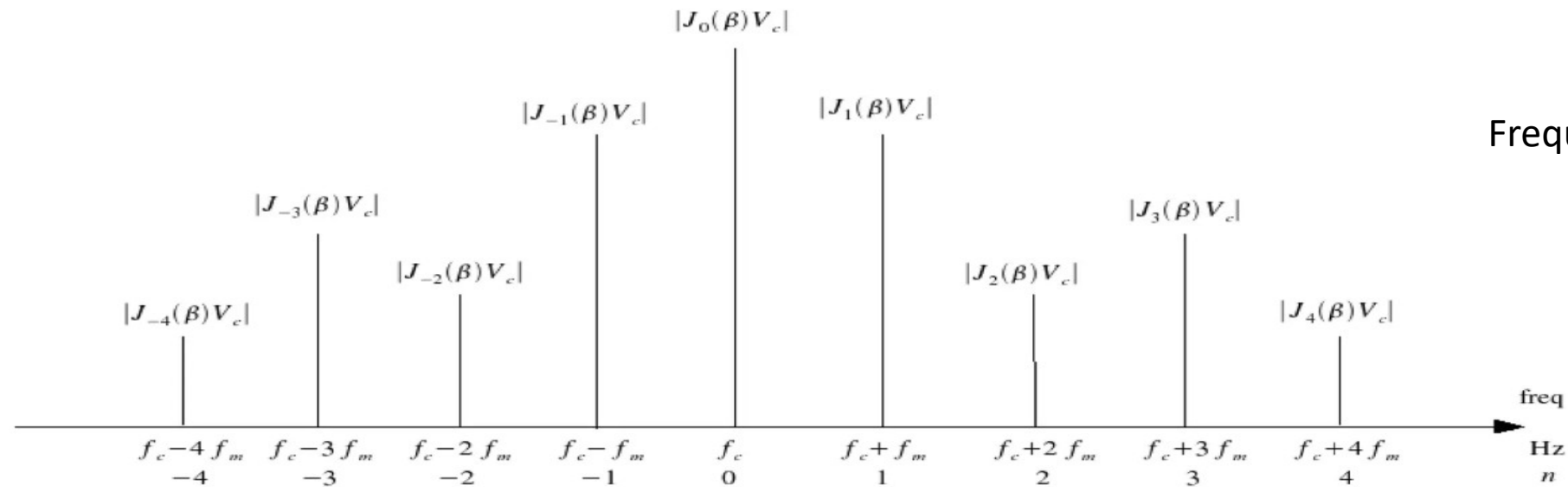
$$s(t) = A_c \cos 2\pi f_c t + A_c J_1(\beta) [\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t] + \\ A_c J_2(\beta) [\cos 2\pi(f_c + 2f_m)t - \cos 2\pi(f_c - 2f_m)t] + \\ A_c J_3(\beta) [\cos 2\pi(f_c + 3f_m)t - \cos 2\pi(f_c - 3f_m)t + \dots]$$

From the above equation, we observe the

1. The expression contains carrier term $\cos 2\pi f_c t$ having magnitude $A_c J_0(\beta)$. This means that the carrier term is reduced by a factor $J_0(\beta)$.
2. From the above expression, it may be noted that in FM, theoretically an infinite number of sidebands are produced and **the amplitude of each sideband** is determined by the **corresponding Bessels function $J_n(\beta)$** . Thus the presence of an infinite number of sideband components makes the **Ideal bandwidth** for FM signal **infinite**. But practically the distinct **sideband with small amplitudes are ignored** and sidebands with significant amplitudes are considered to calculate the bandwidth of FM signal. Thus **practical bandwidth** for FM is **finite**.
3. The number of **significant sidebands generated** in FM **depends** on the value of **modulation Index ' β '**. The modulation index determines how many sideband components have significant amplitudes. This means the practical bandwidth of FM system depends on the value of modulation index.



The amplitude of side frequency components depends upon the Bessel function. The Bessel function variations as a function of ' β ' for fixed value of ' n ' is shown below.



Frequency Spectrum

transmission bandwidth of FM waves:

practical transmission bandwidth of FM wave can be obtained by using the table of Bessels function of first kind

Modulation index	Carrier J_0	Sidebands									
		J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
0.0	1.00	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.06	0.02	—	—
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—
7.0	0.30	0.00	-0.30	-0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02
8.0	0.17	0.23	-0.11	-0.29	0.10	0.19	0.34	0.32	0.22	0.13	0.06

as observed that, as the modulation index increases more and more number of sidebands acquire significant amplitude and the bandwidth is increased.

Therefore, for large values of modulation index ' β ' the bandwidth is slightly greater than the total frequency deviation, $2\Delta f$.

$$\text{Bandwidth (BW): } f_c + \Delta f - (f_c - \Delta f) = 2 \Delta f$$

On the other hand, for small values of modulation index, i.e. $\beta < 0.3$, the spectrum of FM wave is effectively limited to the carrier frequency f_c and one pair of side frequencies at $f_c \pm f_m$.

The bandwidth in this case $2f_m$, $B.W = 2f_m$

Thus we may define an approximate rule for the transmission bandwidth of an FM wave generated is

Transmission bandwidth $B.T = 2\Delta f + 2f_m$

$$= 2\Delta f (1 + f_m/\Delta f)$$

$$= 2\Delta f (1 + 1/\beta)$$

This relation is known as Carson's rule. This equation is the approximate transmission bandwidth.

To obtain the accurate assessment of the bandwidth requirement of an FM wave, we will consider maximum number of significant side frequencies whose amplitudes are greater than 1% of the amplitude of unmodulated carrier.

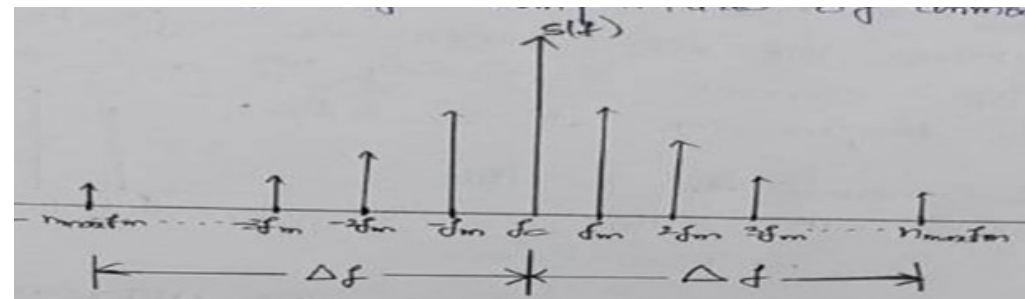
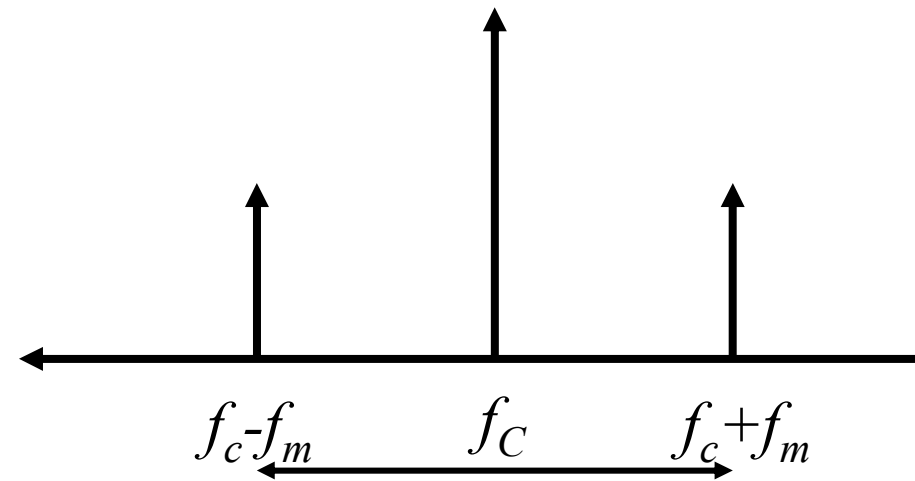
$$\text{Bandwidth (B.W)} = n_{\max} f_m - (-n_{\max} f_m) = 2 n_{\max} f_m$$

In this case the actual transmission bandwidth $(B_T) = 2 n_{\max} f_m$

It is possible to determine if a particular FM signal will be wideband or narrow band by looking at the quality called Deviation Ratio.

Deviation Ratio: It is defined as the ratio of maximum deviation of FM signal to the maximum frequency of modulating signal.

$$\text{Deviation Ratio (DR)} = \Delta f_{\max} / f_m$$



Wide Band Frequency Modulator

WBFM:

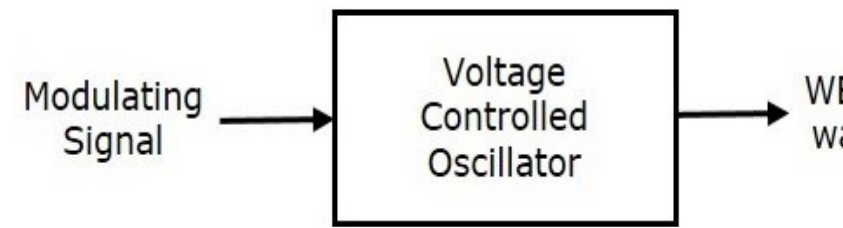
Wide Band Frequency Modulator

Generation of WBFM:

The following two methods generate WBFM wave.

- Direct method
- Indirect method

Direct Method:



This method is called as the Direct Method because we are generating a wide band FM wave directly. In this method, Voltage Controlled Oscillator (VCO) is used to generate WBFM. VCO produces an output signal, whose frequency is proportional to the input signal voltage. This is similar to the definition of an FM wave. The block diagram of the generation of WBFM wave is shown in the figure.

Here, the modulating signal $m(t)$ is applied as an input of Voltage Controlled Oscillator (VCO). VCO produces an output, which is nothing but the WBFM.

$$f_i \propto m(t)$$

$$\Rightarrow f_i = f_c + k_f m(t)$$

f_i is the instantaneous frequency of WBFM wave

Wide Band Frequency Modulator

the reverse direction; the larger the reverse voltage applied to such a diode, the smaller the transition capacitance of the diode. The frequency of oscillation of a Hartley oscillator of Fig. 4.12 is given by

$$f_o(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}$$

where $C(t)$ is the total capacitance of the fixed capacitor and the variable capacitor, and L_1 and L_2 are the two inductances in the frequency-determining network. Assume that for a sinusoidal modulating wave of frequency f_m , the capacitance $C(t)$ is expressed as follows

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t)$$

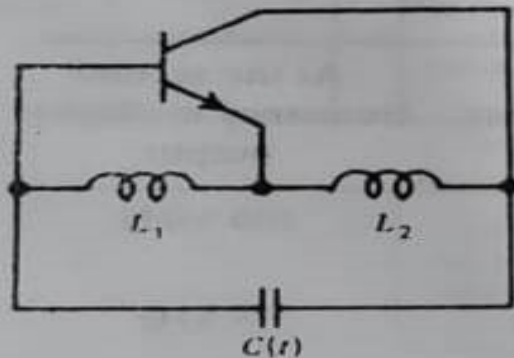


Figure 4.12 Hartley oscillator.

Wide Band Frequency Modulator

CALL NO.

where C_0 is the total capacitance in the absence of modulation and ΔC is the maximum change. Substituting Eq. (4.49) in (4.48), we get

$$f_i(t) = f_0 \left[1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) \right]^{-1/2} \quad (4.50)$$

where f_0 is the unmodulated frequency of oscillation, that is,

$$f_0 = \frac{1}{2\pi\sqrt{C_0(L_1 + L_2)}} \quad (4.51)$$

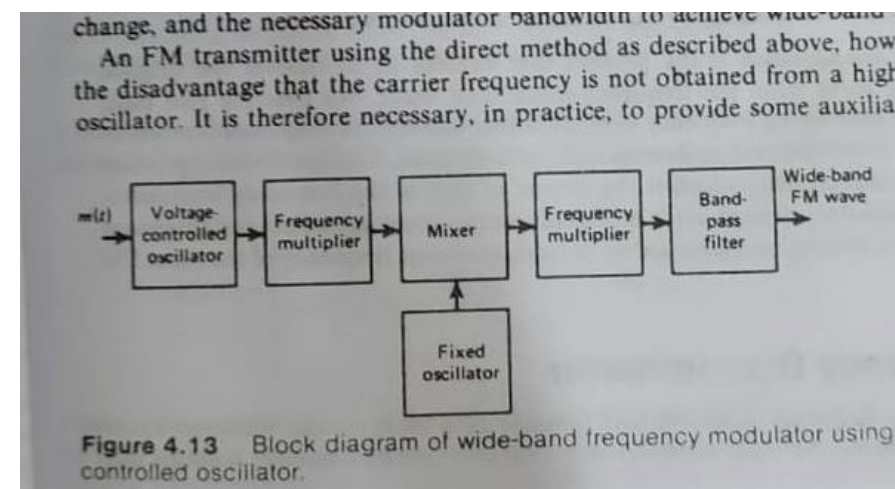
provided that the maximum change in capacitance ΔC is small compared with the unmodulated capacitance C_0 , we may approximate Eq. (4.50) as follows

$$f_i(t) \approx f_0 \left[1 - \frac{\Delta C}{2C_0} \cos(2\pi f_m t) \right] \quad (4.52)$$

Then, by defining

$$\frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_0} \quad (4.53)$$

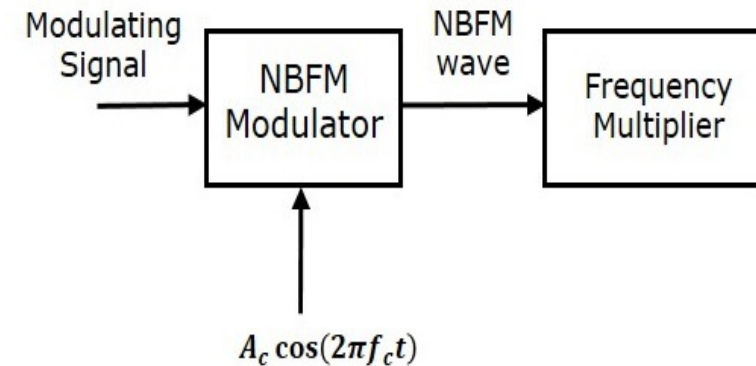
we obtain, for the instantaneous frequency of the oscillator, which is being frequency modulated by varying the capacitance of its frequency-determining resonant network, the following relation

$$f_i(t) \approx f_0 + \Delta f \cos(2\pi f_m t) \quad (4.54)$$


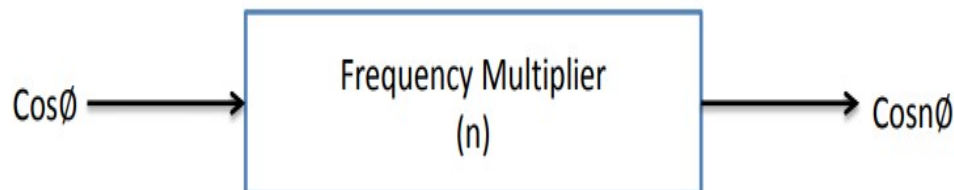
WBFM

Indirect Method

This method is called as Indirect Method because we are generating a wide band FM wave indirectly. This means, first we will generate NBFM wave and then with the help of frequency multipliers we will get WBFM wave. The block diagram of generation of WBFM wave is shown in the following figure.



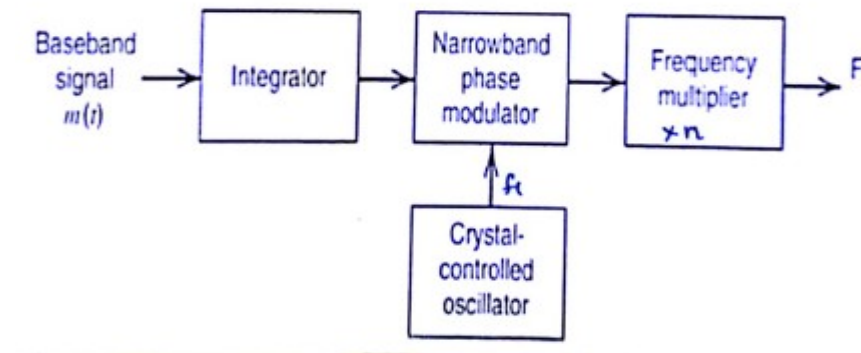
This block diagram contains mainly two stages. In the first stage, the NBFM wave will be generated using NBFM modulator. We have seen the block diagram of NBFM modulator at the beginning of this chapter. We know that the modulation index of NBFM wave is less than one. Hence, in order to get the required modulation index (greater than one) of FM wave, choose the frequency multiplier value properly.



WBFM

This Indirect method is proposed by Armstrong, it is called Armstrong Method. It also called as Stereo FM Method.

The baseband Signal is first integrated by integrator and then applied to 'phase modulator'. The carrier with high frequency stability is generated by crystal oscillator.



Phase modulator generates NBFM.

The NBFM applied to frequency multiplier. Using frequency multiplier and mixer, the required frequency deviation and modulation Index is obtained to generate WBFM.

In order to minimize the distortion in phase modulator, the maximum phase deviation and modulation index kept 1.

Let output of phase modulator is NBFM

$$s_1(t) = A_c \cos(2\pi f_1 t + 2\pi k_1 \int m(t) dt)$$

f_1 frequency of oscillator

k_1 is constant, frequency sensitivity: hz/volts

Consider single tone modulating signal $m(t) = A_m \cos 2\pi f_m t$ Substitute above equation

$$s_1(t) = A_c \cos(2\pi f_1 t + \beta_1 \sin(2\pi f_m t))$$

β_1 is modulation index, kept at 0.3 for minimize the distortion

WBFM

Now phase modulator output is next multiplied by 'n' times in frequency by frequency multiplier.

Frequency Multiplier produces Wide Band FM signal. It becomes

$$s_{\text{WBFM}}(t) = A_c \cos(2\pi n f_1 t + n \beta_1 \sin(2\pi f_m t))$$

$$s_{\text{WBFM}}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$n f_1 = f_c \qquad n \beta_1 = \beta$$

The wide band FM is also written as

$$s_1(t) = A_c \cos(2\pi n f_1 t + 2\pi n k_1 \int m(t) dt)$$

$$s_1(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t) dt)$$

$$n f_1 = f_c \qquad n k_1 = k_f$$

WBFM

Indirect Generation of WBFM for practical Use: (Armstrong Method)

It is a commercial generation of WBFM signal.
For Commercial use it is required to transmit Audio signal with frequency range 50hz to 15khz.
Let the final carrier of FM required $f_c = 100\text{MHz}$

Consider a NBFM with carrier frequency $f_{c1} = 100\text{KHz}$ generated by the crystal oscillator.

To limit the harmonic distortion of narrow band phase modulator, we restrict the modulation index is 0.3

Let $\beta_1 = 0.2$ radians

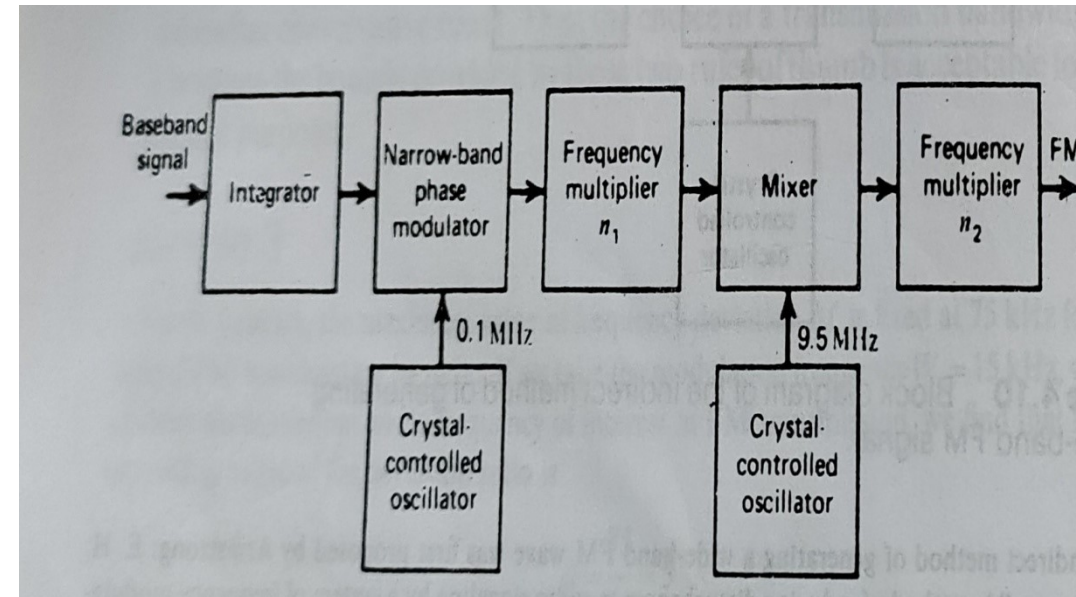
For lowest modulating frequency 50HZ

$$\Delta f_1 = \beta_1 \times f_{m1} = 0.2 \times 50 = 10\text{Hz}$$

For highest modulating frequency 15kHz

$$\Delta f_2 = \beta_1 \times f_{m2} = 0.2 \times 15\text{K} = 3\text{KHz}$$

It is the frequency deviation at the output of phase modulator



WBFM

In order to produce a frequency deviation of $\Delta f = 75 \text{ kHz}$ at the output with $\Delta f_1 = 10 \text{ Hz}$ at the output of phase modulator

The frequency multiplier should multiply with total frequency multiplication factor

$$n = 75000 / 10 = 7500$$

This multiplication factor is placed in two multipliers as shown in above figure with $n_1 = 100$ and $n_2 = 75$

Mixer is a frequency translation network. It is translate the frequency down or up without altering Δf .

Mixer produces the required frequency f_c

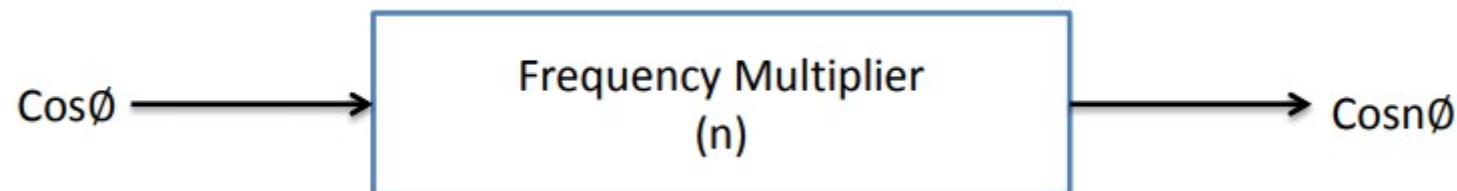
WBFM

Frequency multiplier: is a device for increasing the deviation. It would have been better to have called it a deviation multiplier. It does indeed multiply the frequency, but this is of secondary importance in this application.

Frequency multiplier is a non-linear device, which produces an output signal whose frequency is 'n' times the input signal frequency. Where, 'n' is the multiplication factor.

If NBFM wave whose modulation index β is less than 1 is applied as the input of frequency multiplier, then the frequency multiplier produces an output signal, whose modulation index is 'n' times β and the frequency also 'n' times the frequency of WBFM wave.

Sometimes, we may require multiple stages of frequency multiplier and mixers in order to increase the frequency deviation and modulation index of FM wave.



WBFM

- When NBFM is passed through Frequency multiplier WBFM is obtained.
- The output of the NBFM oscillator is
- $S_{NBFM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$
- Where β is very less than 1
- The output of frequency multiplier with multiplying factor n is a wideband FM that is
- $S_{WBFM}(t) = A_c \cos[n(2\pi f_c t + \beta \sin 2\pi f_m t)]$
- $\therefore S_{WBFM}(t) = A_c \cos[2\pi n f_c t + n\beta \sin 2\pi f_m t]$
- $\therefore S_{WBFM}(t) = A_c \cos[2\pi f_c' t + \beta' \sin 2\pi f_m t]$

Where $f_c' = n f_c$

And $\beta' = n\beta > 1$

£

Input of frequency multiplier (n)	Output
f_c	$n f_c$
β	$n\beta$
f_m	f_m
$\Delta f = \beta f_m$	$n\Delta f$

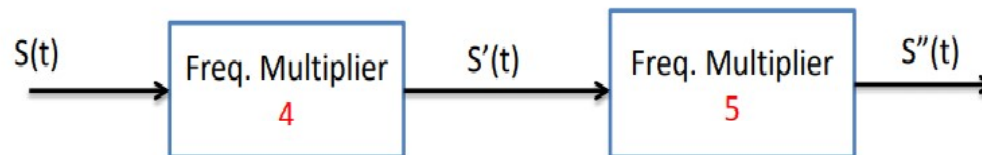
WBFM

Q. An FM is given by:

$$S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin 4\pi \times 10^3 t]$$

It is passed through cascaded frequency multiplier having multiplying constant of 4 and 5 respectively.

Find all the parameters of FM signal at the output of each of the multiplier. (A_c , β , f_c , f_m , Δf , bandwidth, power)



$$S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin 4\pi \times 10^3 t]$$

- Compare with the standard equation

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

- $A_c = 10V$, $\beta = 0.2$, $f_c = 1MHz$, $f_m = 2KHz$, $\Delta f = \beta f_m = 0.4KHz$

After passing through first multiplier ($n=4$)

- $A_c' = 10V$
- $\beta' = 4 \times 0.2 = 0.8$ (NBFM)
- $f_c' = 4 \times 1 = 4MHz$
- $f_m' = 2KHz$ (No change)

After passing through second multiplier ($n=5$)

- $A_c'' = 10V$
- $\beta'' = 5 \times 0.8 = 4$ (WBFM)
- $f_c'' = 5 \times 4 = 20MHz$
- $f_m'' = 2KHz$ (No change)
- $\Delta f'' = n \Delta f' = 5 \times 1.6KHz = 8KHz$
- $BW = 2(1 + \beta) f_m = 2 \times 5 \times 2 = 20KHz$
- Power = $\frac{A_c^2}{2R} = 50W$

FM Demodulators

The process of extracting an original message signal from the FM modulated wave is known as **detection** or **demodulation**. The circuit, which demodulates the FM modulated wave is known as the FM **demodulator** or **Detectors**. Let us discuss about the FM demodulators which demodulate the FM wave.

- ☐ Demodulation process of FM waves is exactly **opposite** to that of frequency modulation
- ☐ **FM demodulator** operates on **different principle** than **AM detector**
- ☐ **FM demodulator** is basically a **frequency to amplitude converter** i.e. converting the frequency variations in FM wave at its input into amplitude variations at its output to recover original modulating signal

Some requirements of FM Demodulator/Detector are:

- It must **convert frequency variations** into amplitude variations
- Conversion must be **linear**
- Conversion must be **efficient**
- Demodulator circuit must be **insensitive to amplitude changes** i.e. it must respond only to frequency variations
- Its **operation** and **adjustment** must not be too critical

FM Demodulators

The process of extracting an original message signal from the FM modulated wave is known as **detection** or **demodulation**. The circuit, which demodulates the FM modulated wave is known as the **FM demodulator or Detectors**. Let us discuss about the FM demodulators which demodulate the FM wave.

Classification of FM demodulators :

1. Direct Method -
 - 1.1: Frequency discrimination method :- 1. Simple Slope Detector and 2. Balanced Slope Detector
 - 1.2: Phase discrimination method :- 1. Foster Seeley Discriminator and 2. Ratio detector
2. Indirect Method - PLL

FM demodulators or detectors can perform the extraction of modulating signal from a modulated signal in two steps.

1. It converts the frequency modulated wave into corresponding Amplitude Modulated (AM) Wave by using a frequency dependent circuit i.e. Circuits whose output voltage depends on input frequency. Such circuits are called frequency discriminators.
2. The original modulating signal $m(t)$ is recovered from this AM signal by using a linear diode envelope detector. A simple RL circuit can be used as a discriminator, but this circuit has a poor sensitivity. Therefore tuned LC circuits are used as frequency discriminators.

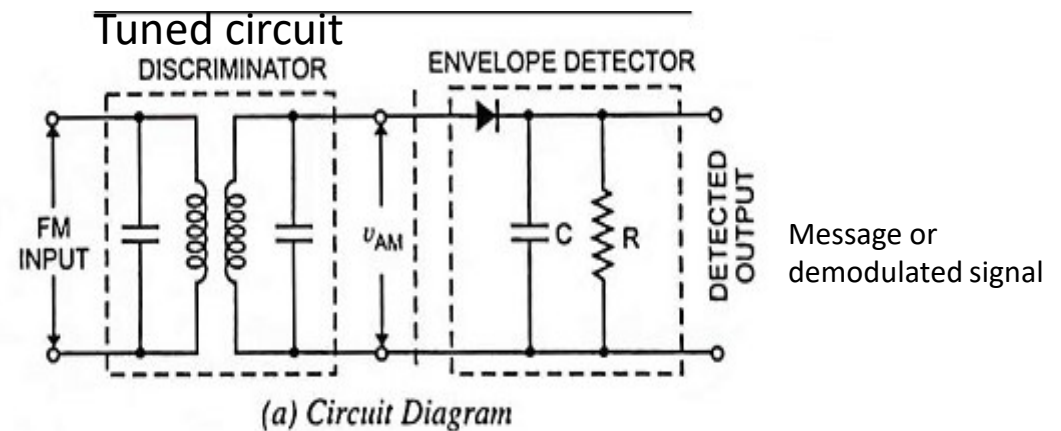
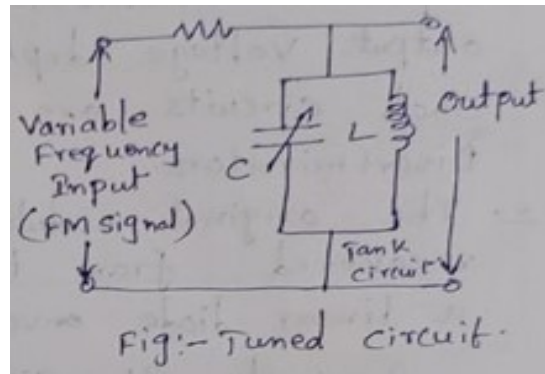
FM Demodulators

Simple Slope Detector: The frequency discriminator operates on the principle of slope detection.

Principle of slope detection: Let us consider tuned circuit shown in figure.

A frequency modulated signal is applied to the tuned circuit with centre frequency ' f_c ' and frequency deviation ' Δf '.

The resonant frequency of the tuned circuit is slightly detuned from the carrier frequency ' f_c ' i.e the resonant frequency of the tuned circuit is adjusted to $(f_c + \Delta f)$.



The circuit of simple slope detector is shown above figure. This circuit convert the FM signal into an AM signal i.e. The AM signal is then detected by a diode detector.

A small variation in the frequency of the input signal will produce a change in amplitude of e_{AM} .

FM Demodulators

A slope detector linear frequency to amplitude transfer characteristics for a particular bandwidth.

The output voltage of a tank circuit is a amplitude modulated wave which is then applied to a simple diode detector with proper time constant.

This detector is similar to AM diode detector and output of the detector will be AF modulating signal.

Drawbacks:

It is inefficient.

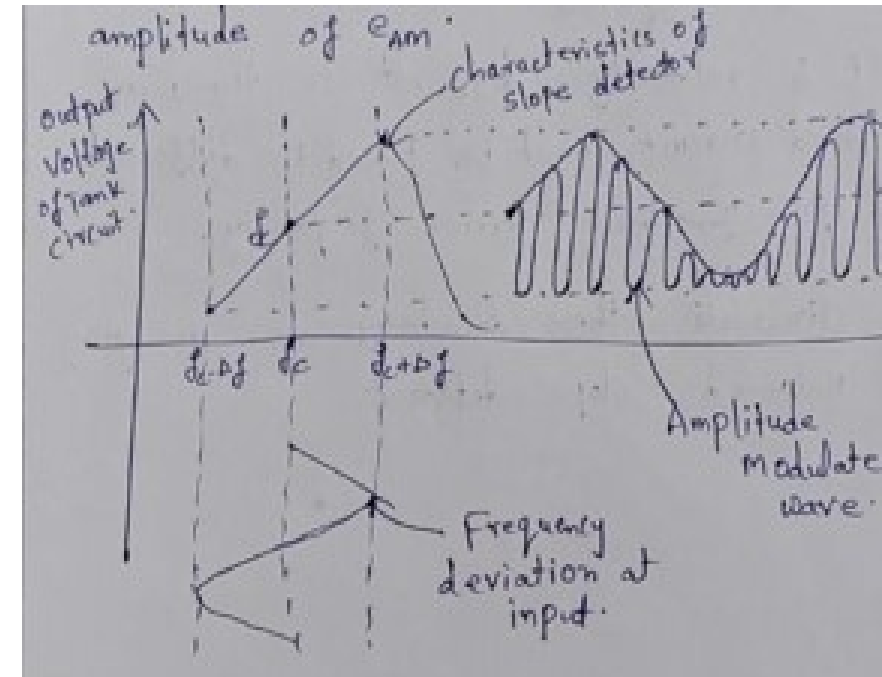
It is linear only over a limited frequency range.

It can operates narrow band of frequencies.

Non linear characteristics of gain – Cannot reproduce exact message signal

It is difficult to Tune the circuits at different tuned frequencies, Circuit complexity is high

To minimize these drawbacks, we are going for Balanced slope detector.



FM Demodulators

Balanced Slope Detector:

- ☒ Demodulation process of FM waves is exactly **opposite** to that of frequency modulation
- ☐ **FM demodulator** operates on **different principle** than **AM detector**
- ☐ **FM demodulator** is basically a **frequency to amplitude converter** i.e. converting the frequency variations in FM wave at its input into amplitude variations at its output to recover original modulating signal

FM Demodulators

Balanced Slope Detector:

A balanced slope detector is an improved version of the slope detector. The drawback of harmonic distortion is removed in this detector by using two slope detectors instead of one as in a single-tuned slope detector.

Circuit consists of two LC tuned circuits T_1 & T_2 and it is the combination of two slope detectors.

The two tuned circuits are in the stagger tuned mode. i.e. One is tuned above the carrier frequency ($f_c + \Delta f$) and other is tuned to below the carrier frequency ($f_c - \Delta f$).

R_1C_1 and R_2C_2 are the fillers used to bypass the RF ripple. V_{01} and V_{02} are the output voltages of the two slope detectors. The final output $V_0 = V_{01} - V_{02}$.

Working Operation of the circuit:

Case 1:

If $f_{in} = f_c$, i.e. When the input frequency is instantaneously equal to f_c , the voltage induced at T_1 and T_2 are exactly equal i.e. $V_{01} = V_{02}$, then $V_0 = 0$

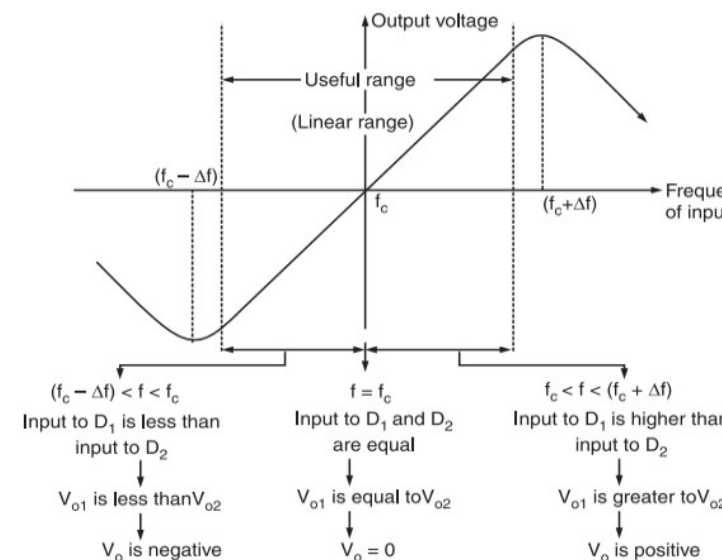
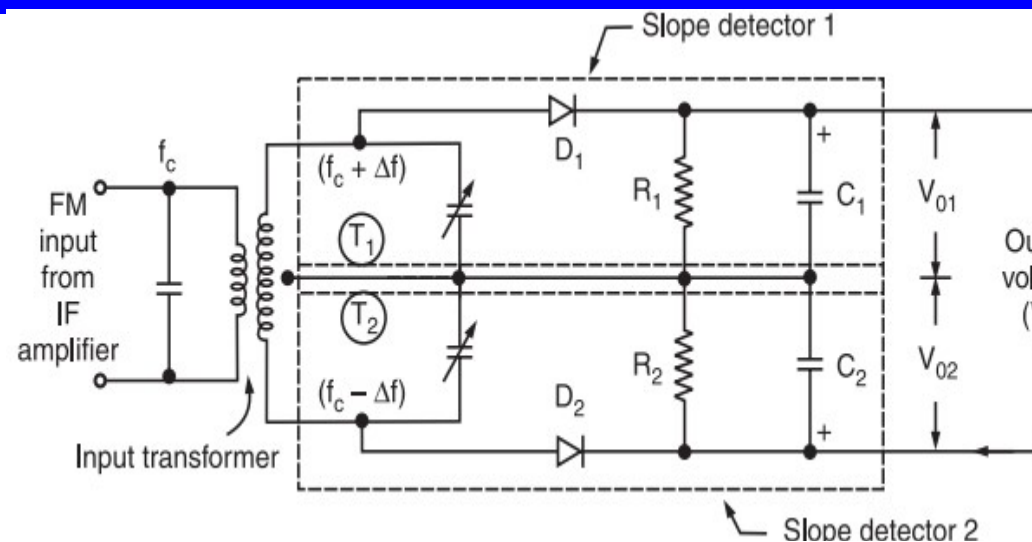


Fig. 2.5.4 Characteristics of the balanced slope detector

FM Demodulators

Case1:

If $f_c < f_{in} < f_c + \Delta f$, i.e. When the input frequency lies between f_c and $f_c + \Delta f$ voltage induced at T1 greater than T2. Therefore input to D1 is higher than D2. Hence V_{01} is higher than V_{02} ($V_{01} > V_{02}$). i.e. Then output V_0 is positive. As input frequency increases towards $f_c + \Delta f$, the output voltage also increases positively.

Case2:

If $f_c - \Delta f < f_{in} < f_c$, i.e. When the input frequency lies between f_c and $f_c - \Delta f$ voltage induced at T1 less than T2. Therefore input to D1 is less than D2. Hence V_{02} is higher than V_{01} ($V_{02} > V_{01}$). i.e. Then output V_0 goes negatively.

Characteristics of Balanced Slope Detector

- ✓ Due to typical shape it is known as "S" shape characteristics
- There is a linear portion at the center of response curve
- Towards the edge the response becomes very distorted

Advantages

- ✓ ☐ This circuit is more efficient in comparison to simple slope detector.
- ✓ ☐ It has better linearity than the simple slope detector

Disadvantages

- ✓ ☐ Even though linearity is good but it is not good enough
- ☐ Difficult to tune this circuit since the three tuned circuits are to be tuned at different frequencies i.e., f_c , $(f_c + \Delta f)$ and $(f_c - \Delta f)$
- ☐ Amplitude limiting is not provided

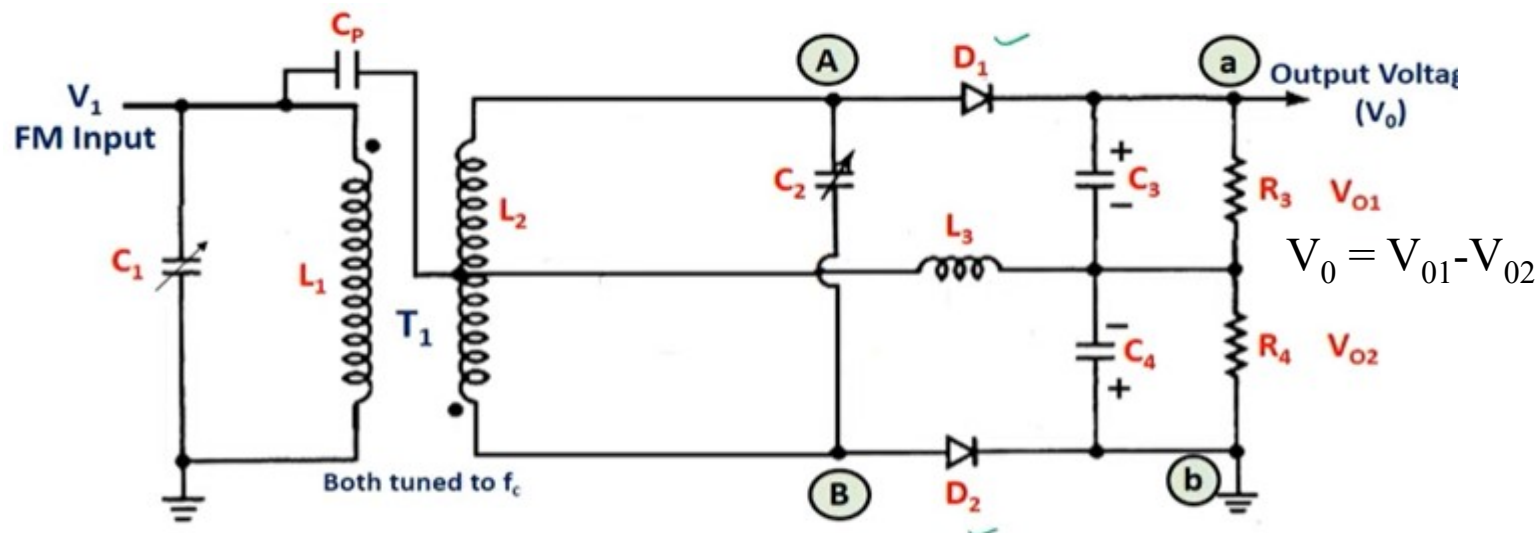
FM Demodulators

Phase discrimination method :- 1.Foster Seeley Discriminator and 2.Ratio detector

Some requirements of FM Demodulator/Detector are:

- It must **convert frequency variations** into amplitude variations
- Conversion must be **linear**
- Conversion must be **efficient**
- Demodulator circuit must be **insensitive to amplitude changes** i.e. it must respond only to frequency variations
- Its **operation** and **adjustment** must not be too critical

1.Foster Seeley Discriminator



Operation:

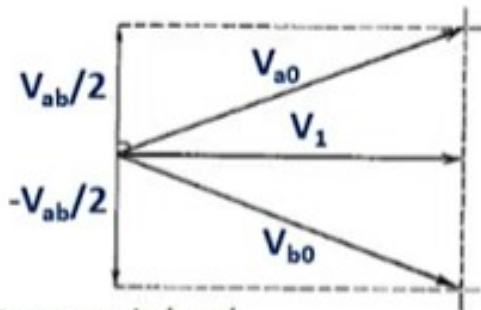
Case 1: If $f_{in} = f_c$
 $V_{O1} = V_{O2}$ $V_0 = 0$

Case 2: If $f_{in} > f_c$
 $V_{O1} > V_{O2}$ $V_0 = P$

Case 3: If $f_{in} < f_c$
 $V_{O1} < V_{O2}$ $V_0 = \text{Neg}$

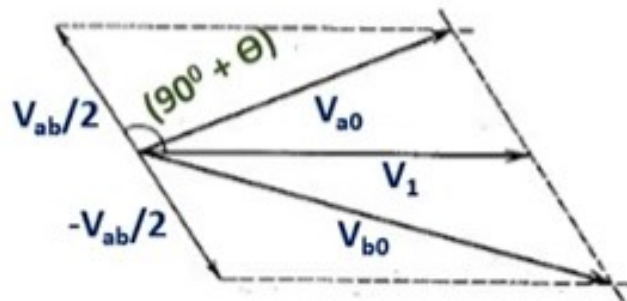
FM Demodulators

Relation between primary & secondary

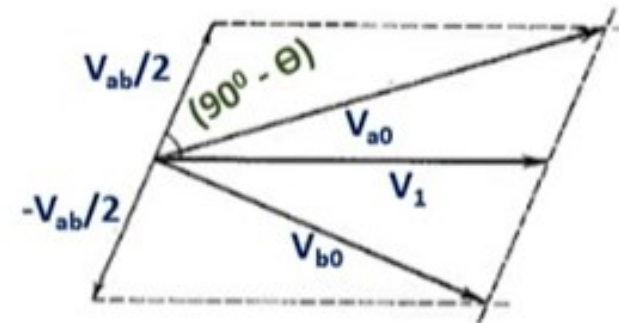


Equal voltages are induced in two halves of secondary

Phasor diagram for $f_{in} = f_c$



Secondary equivalent circuit

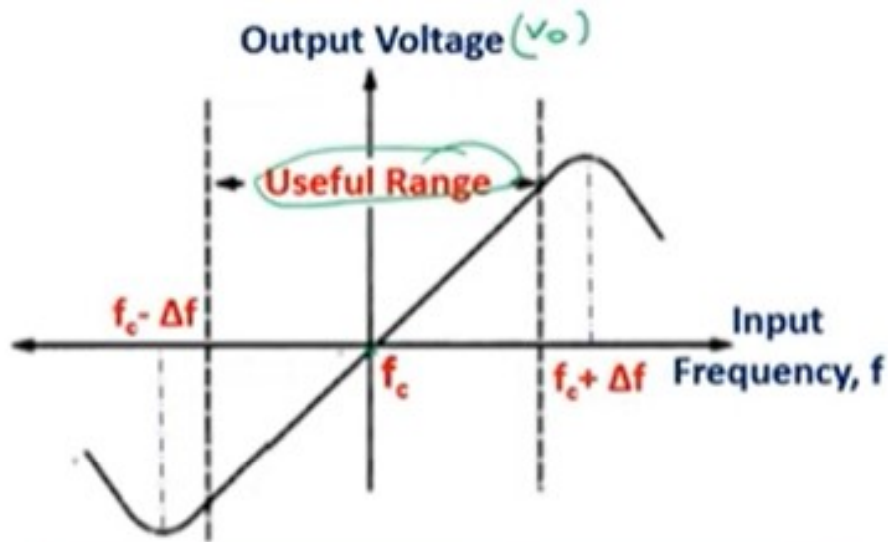


From figure, it is clear that the primary and secondary voltages are:

- Exactly 90° out of Phase Discriminator when input frequency is f_c
- Less than 90° out of Phase Discriminator when f_{in} is higher than f_c
- More than 90° out of Phase Discriminator when f_{in} is below f_c



FM Demodulators



Frequency response of Phase Discriminator

- ✓ The response which is normally seen for an FM demodulator / FM detector is known as an "S" curve
- ✓ There is a **linear portion** at the center of response curve
- ✓ Towards the **edge** the response becomes very **distorted**

Advantages

- ✓ ☐ Offers good performance
- ✓ ☐ Linearity is better
- ✓ ☐ Simple to construct using discrete components
- ✓ ☐ Provides higher output than the ratio detector
- ✓ ☐ Provides more linear output i.e. lower distortion in comparison to ratio detector

Disadvantages

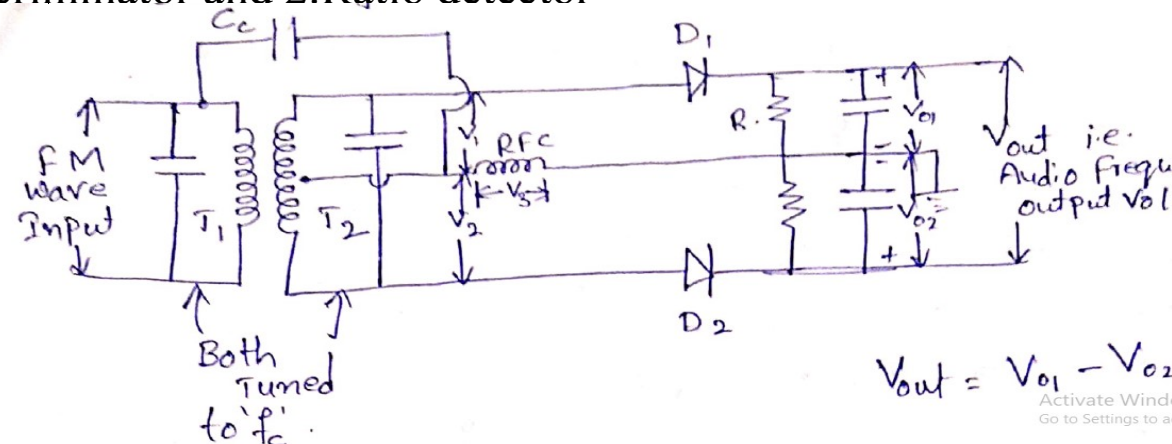
- ✓ ☐ Does not provide amplitude limiting
- ✓ ☐ High cost of transformer
- ✓ ☐ Narrower bandwidth than the ratio detector
- ✓ ☐ The circuit is sensitive to both frequency and amplitude so limiter is required to remove amplitude variations

FM Demodulators

Phase discrimination method :- 1. Foster Seeley Discriminator and 2. Ratio detector

Foster Seeley Discriminator:

Foster Seeley Discriminator look like a balanced slope detector. The only difference is the process of applying the voltage to the diodes.

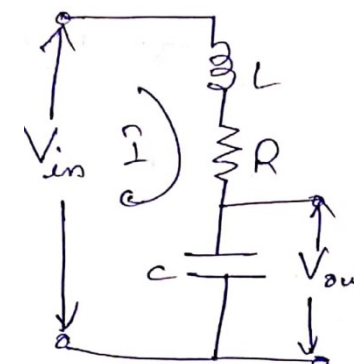


In this circuits two tuned circuits T_1 & T_2 are used which are tuned to the same frequency f_c of the incoming signal. This simplifies the tuning process to a great extent.

Even though the primary and secondary are tuned to the same center frequency (f_c), the voltage applied to the two diodes are not same. This is due to change in phase shift between the primary and secondary windings on the input frequency.

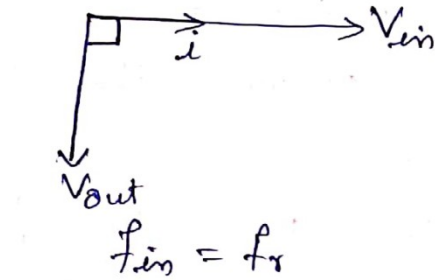
Let us see how this circuit provides frequency dependent phase shifting

Consider series RLC circuit, the output voltage taken across the capacitor, the output voltage ' V_0 ' and current ' I ' are in 90° phase difference.

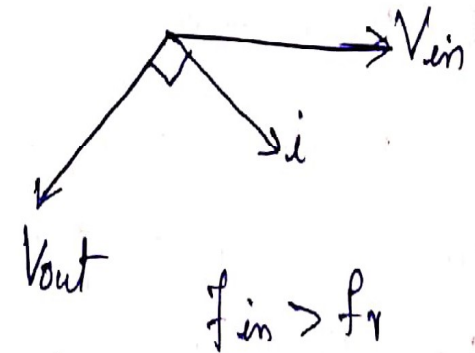


FM Demodulators

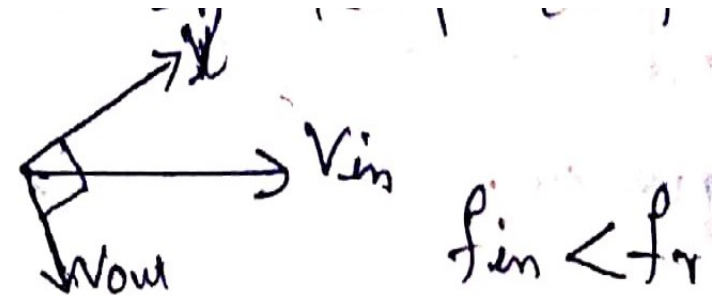
At resonant frequency, the series RLC circuit behaves as purely resistive circuit, hence input voltage and currents are in phase.



At above resonant frequency, the series RLC circuit behaves as inductive circuit, hence input current 'I' lags to input voltage V_{in} .



At below resonant frequency, the series RLC circuit behaves as capacitive circuit, hence input current 'I' leads to input voltage V_{in} .



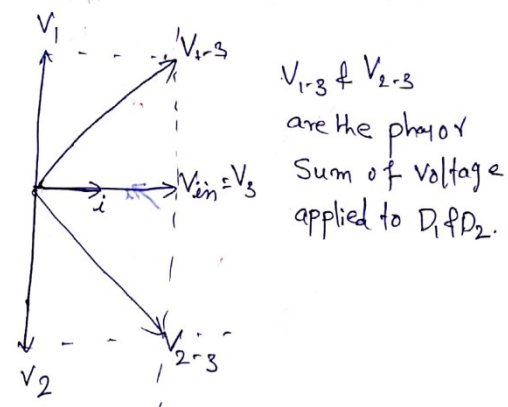
Based on this principle, The Foster Seeley discriminator will work

FM Demodulators

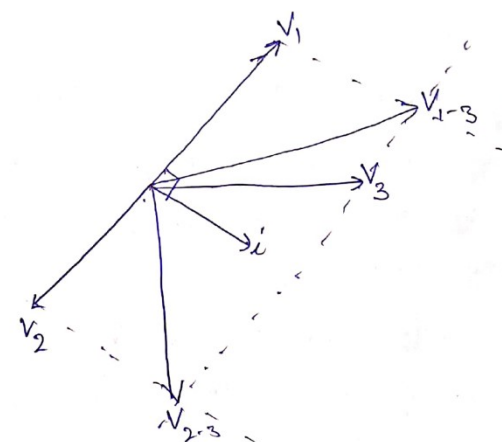
The voltage developed across primary will appear across Radio Frequency Choke through a coupling Capacitor i.e. Voltage across RFC is the primary voltage V_3 .

Therefore phasor sum of V_1 & V_3 i.e. V_{1-3} is applied to diode D1 & phasor sum of V_2 & V_3 i.e. V_{2-3} will be applied to diode D2.

Case 1: If $f_{in} = f_c$, at resonance, the circuit is purely resistive. At this time, the voltage applied to the diodes are same but 180° out of phase and the resultant output voltage is zero.

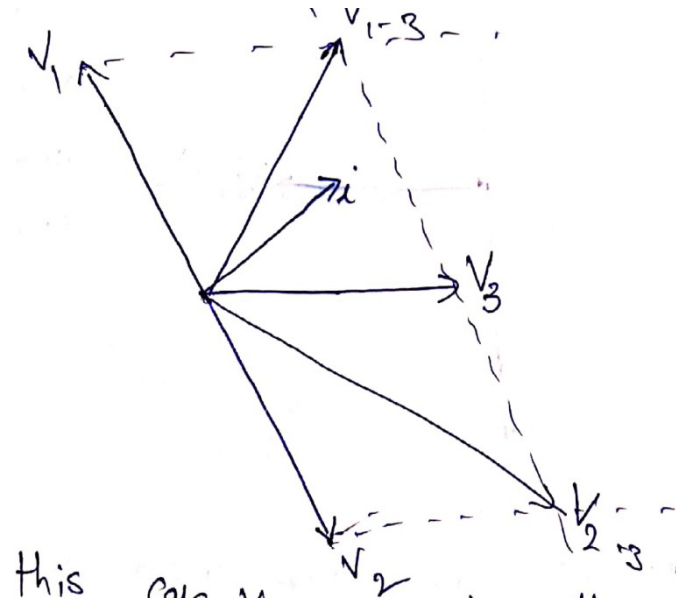


Case 2: If $f_{in} > f_c$, above resonance, the circuit is inductive and current lags compared to $V_{in} = V_3$. Due to this, there will be a phase shift of less than 90° in V_1 as shown in the fig. In this case, V_{1-3} is higher than V_{2-3} , hence the voltage applied to diode D1 is more than to D2. Hence the output voltage is positive.



FM Demodulators

Case 3: If $f_{in} < f_c$, below resonance, the circuit is capacitive and current leads compared to $V_{in} = V_3$. Due to this, there will be phase shift of greater than 90° in V_1 is shown in fig. In this case, V_{1-3} is less than V_{2-3} , hence voltage applied to diode D_1 is less than D_2 . Hence output voltage is negative.



Advantages of Foster-Seeley FM discriminator:

1. Offers good level of performance and reasonable linearity.
2. Simple to construct using discrete components.
3. Provides higher output than the ratio detector

Disadvantages of Foster-Seeley FM discriminator:

1. Does not easily lend itself to being incorporated within an integrated circuit.
2. High cost of transformer.
3. Narrower bandwidth than the ratio detector

FM Demodulators

Ratio Detector:

The Foster Seeley discriminator has the disadvantage that any variation in amplitude of the input FM signal due to noise, modifying the characteristics of discriminator.

The undesired frequency components corresponding to amplitude variations are produced in the detected output and the output gets distorted.

This distortion is reduced using a limiter circuit in the FM receivers.

Ratio Detector is an improvement over the Foster – Seeley discriminator and is widely used. It does not respond to amplitude variations, a limiter is not needed. The circuit is shown

Ratio Detector is similar to the Foster Seeley discriminator except the following changes

1. The detector diode D_2 is reversed.
2. A large value capacitor C_5 has been included in the circuit
3. The output is taken some where else.

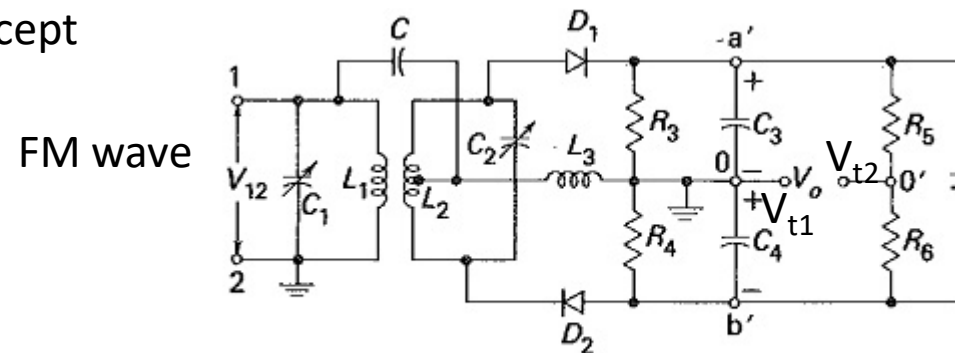
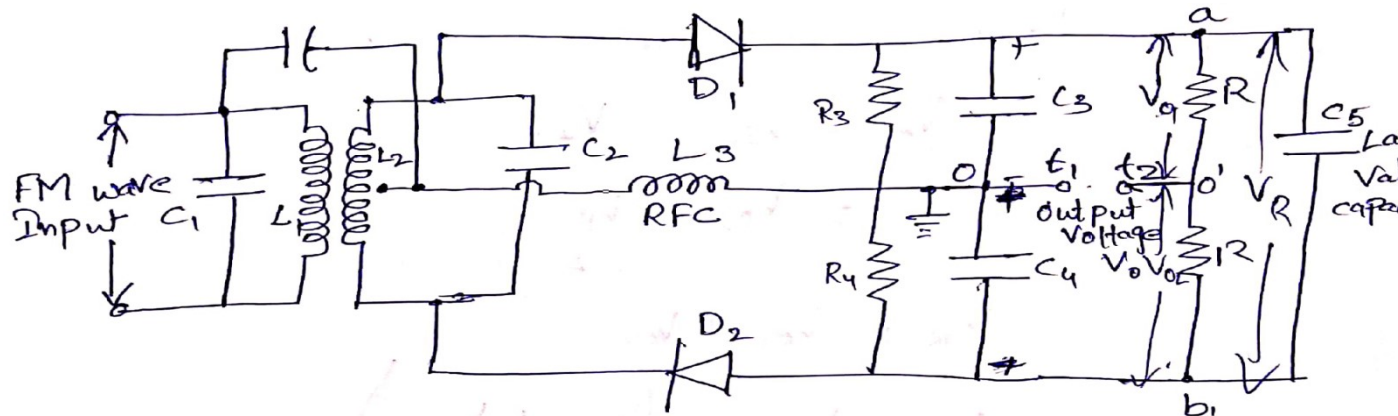


FIGURE 6-41 Basic ratio detector circuit.

FM Demodulators

Amplitude limiting in Ratio detector: If V_i at input increases, secondary voltage of transformer increases, so current increases, across the load (R_5+R_6) Capacitor C_5 connected. If current increases, voltage across capacitor change instantaneously, it increases gradually. Voltage across C_5 is constant. Load impedance decreases. T Secondary of transformer heavily damped. Because of heavily damping, Quality(Q) factor of transformer decreases. Causing the gain of the amplifier is decreases. Similar process repeat v_i is decreases.



$$V_0 = V_{t1} - V_{t2}$$

$$V_0 = V_{02} - \frac{V_R}{2}$$

$$V_R = V_{01} + V_{02}$$

$$V_0 = V_{02} - \frac{(V_{01} + V_{02})}{2}$$

$$V_0 = \frac{(V_{02} - V_{01})}{2}$$

FM Demodulators

Operation of Ratio Detector : The ratio detector output voltage is equal to half of the difference between the output voltages from the individual diodes.

Hence, Similar to phase discriminator, the output voltage is proportional to the difference between individual outputs.

Due to this reason, the operation of Ratio Detector is identical to the phase discriminator and phasor diagrams are also similar to the Foster Seeley discriminator.

The additional feature of the Ratio Detector is the amplitude limiting action is included due to addition of large capacitor C_5 .

$$V_0 = V_{t1} - V_{t2}$$

$$V_0 = V_{02} - \frac{V_R}{2}$$

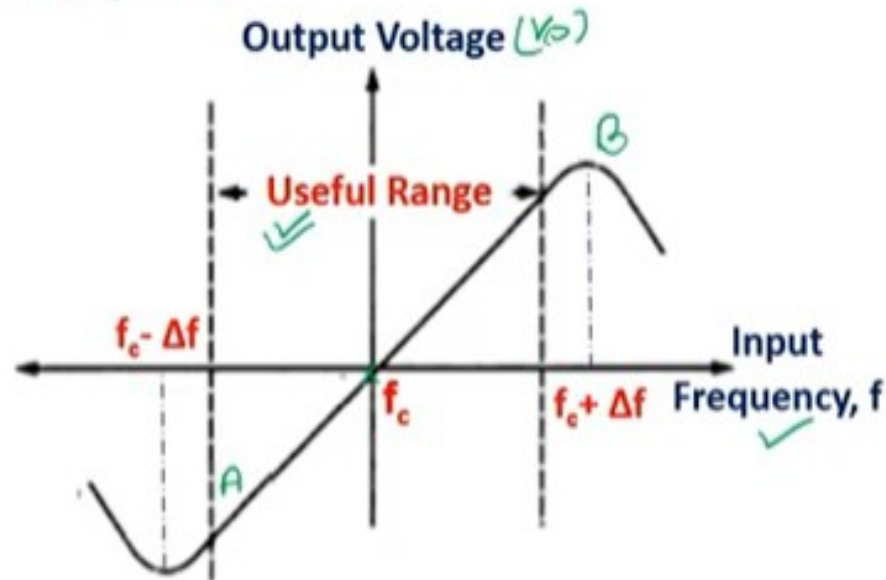
$$V_R = V_{01} + V_{02}$$

$$V_0 = V_{02} - \frac{(V_{01} + V_{02})}{2}$$

$$V_0 = \frac{(V_{02} - V_{01})}{2}$$

FM Demodulators

$$V_o = V_{o1} - V_{o2}$$



Frequency response

Advantages

- ✓ ☐ Easy to align
- ✓ ☐ Good linearity due to linear phase relationship between primary and secondary
- ✓ ☐ Amplitude limiting is provided inherently so additional limiter is not required

- ✓ The response which is normally seen for an FM demodulator / FM detector is known as an "S" curve
- There is a linear portion at the center of response curve
- Towards the edge the response becomes very distorted

FM Demodulators

Phase Locked Loop (PLL):

A phase Locked Loop(PLL) is used in tracking the phase and frequency of the carrier component of an incoming FM signal.

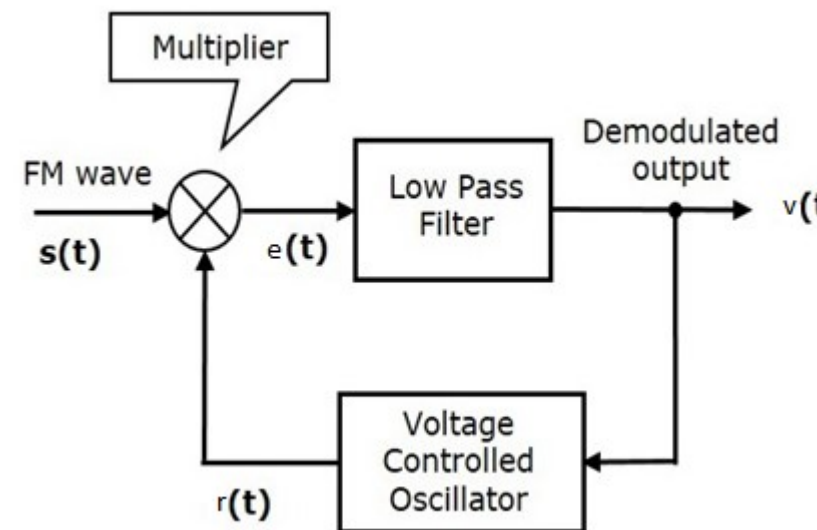
PLL is used for demodulating the FM signals in presence of large noise and low signal power. So PLL is used in Commercial FM receivers.

In PLL negative feedback system presents and consists of the multiplier, the low pass filter and the Voltage Controlled Oscillator (VCO). VCO produces an output signal $r(t)$, whose frequency is proportional to the input signal voltage.

The operation of PLL is similar to any other feedback system. In any feedback system, the feedback signal tends to follow the input signal.

If the feedback signal is not equal to input signal, an error signal will exist. This error signal will change the value of feedback signal, until it is equal to the input signal.

The difference between $s(t)$ and $r(t)$ is called an error signal. The error signal is used to adjust the VCO frequency in such a way that the instantaneous phase angle come close to angle of incoming signal $s(t)$. At this point, the two signals $s(t)$ and $r(t)$ are in synchronized and the PLL is locked to the incoming signal $s(t)$.



FM Demodulators

A VCO is a sine wave generator whose frequency is determined by the voltage applied to it from external source means that any frequency modulator can work as a VCO.

The following mathematical analysis would help to understand how the FM demodulator or detector can be performed using PLL.

Here, we have assumed that the VCO is adjusted initially so that control voltage comes to zero, the following conditions are satisfied.

- i) The frequency of the VCO is precisely set at the unmodulated carrier frequency ' f_c ' and
- ii) The VCO output has a 90° phase shift with respect to the unmodulated carrier.

Let input signal applied to the phase locked loop is an FM wave is defined by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \quad \text{---(1)} \quad A_c \text{ is amplitude of carrier signal}$$

Phase of the modulator

$$\phi_1(t) = 2\pi k_f \int_0^t m(t) dt \quad \text{---(2)} \quad k_f \text{ is frequency sensitivity of modulator}$$

FM Demodulators

Let VCO output is defined by

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)] \quad \text{---(3)}$$

Where A_v is the amplitude of the output voltage of VCO

$$\phi_2(t) = 2\pi k_v \int_0^t v(t) dt \quad \text{---(4)}$$

Where $v(t)$ is the control voltage applied VCO input.

k_v is frequency sensitivity of VCO (Hz / volts)

The incoming FM wave $s(t)$ and VCO output $r(t)$ are applied to the multiplier, producing two components

$$\begin{aligned} i. e. \quad s(t)r(t) &= A_c \sin[2\pi f_c t + \phi_1(t)] A_v \cos[2\pi f_c t + \phi_2(t)] \\ &= K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)] \\ &\quad - K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] \end{aligned}$$

From the above equation high frequency components represented as $K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$

Low frequency components represented as $K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$ K_m is multiplier gain (V^{-1})

High frequency component is eliminated by Low Pass Filter. Therefore discarding high frequency component

FM Demodulators

The input to the loop filter (LPF) is now given by

$$e(t) = K_m A_c A_v \sin[\phi_e(t)] \quad \text{---(5)}$$

Where $\phi_e(t)$ is phase error is $\phi_e(t) = \phi_1(t) - \phi_2(t)$

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t v(t) dt \quad \text{---(6)}$$

The output produced by the loop filter is $V(t) = \int_{-\infty}^{\infty} e(\tau) h(t - \tau) d\tau \quad \text{---(7)}$

Relate $\phi_e(t)$ and $\phi_1(t)$ using equations (5),(6),(7)

$$\begin{aligned} \frac{d\phi_e(t)}{dt} &= \frac{d\phi_1(t)}{dt} - 2\pi k_v \int_{-\infty}^{\infty} K_m A_c A_v \sin\phi_e(\tau) h(t - \tau) d\tau \\ &= \frac{d\phi_1(t)}{dt} - 2\pi K_m K_v A_c A_v \int_{-\infty}^{\infty} \sin\phi_e(\tau) h(t - \tau) d\tau \end{aligned}$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_0 \int_{-\infty}^{\infty} \sin\phi_e(\tau) h(t - \tau) d\tau \quad \text{---(8)}$$

$K_0 = K_m K_v$

The amplitudes A_c & A_v measured in volts, multiplier gain Volt-1, frequency sensitivity is Hz / volts, hence K_0 the dimensions of frequency.

FM Demodulators

When the phase error $\phi_e(t)$ is zero, the phase locked loop is said to be in phase lock.

When the phase error $\phi_e(t)$ is small compared to one radian,

We may use approximation $\sin\phi_e(t) \approx \phi_e(t)$

In this case loop is said to be near phase lock

Equation(8) becomes

$$\frac{d\phi_e(t)}{dt} + 2\pi k_0 \int_{-\infty}^{\infty} \phi_e(\tau) h(t - \tau) d\tau = \frac{d\phi_1(t)}{dt} \quad \text{---(9)}$$

On the basis of above equation, we can construct the equivalent model of PLL as shown in below figure

To convert frequency domain, taking the Fourier transform of equation (9)

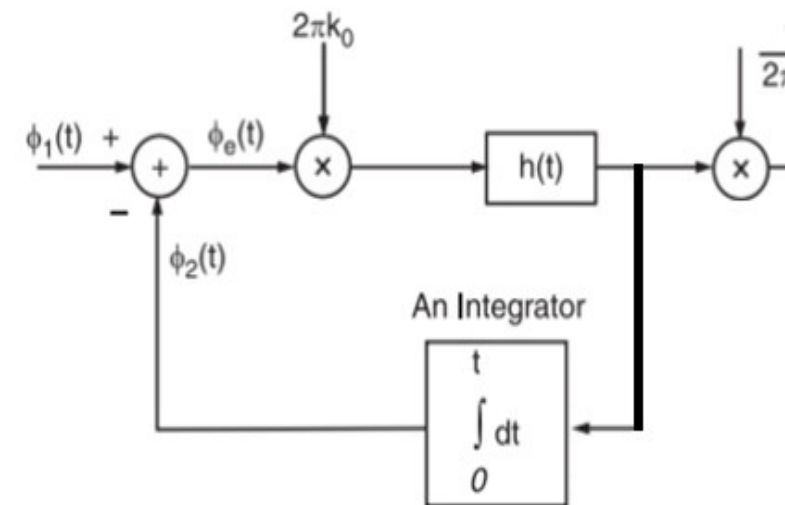


Figure. 2.6.3 Equivalent model of PLL

FM Demodulators

To convert frequency domain, taking the Fourier transform of equation (9)

$$J\omega\phi_e(f) \left[1 + \frac{2\pi K_0 H(f)}{J\omega} \right] = J\omega\phi_1(f)$$

$$\phi_e(f) \left[1 + \frac{2\pi K_0 H(f)}{J2\pi f} \right] = \phi_1(f)$$

$$\phi_e(f) \left[1 + \frac{K_0 H(f)}{Jf} \right] = \phi_1(f)$$

$$\phi_e(f) = \left[\frac{\phi_1(f)}{1 + \frac{K_0 H(f)}{Jf}} \right]$$

$$\phi_e(f) = \left[\frac{\phi_1(f)}{1 + L(f)} \right] \quad \text{---(10)}$$

Where $L(f)$ is called open loop transfer function of Phase Locked Loop is given by $L(f) = \frac{K_0 H(f)}{Jf}$ ---(11)

FM Demodulators

From the liberalised model, we see that $V(f)$ is the Fourier transform of PLL output is related to $\phi_e(f)$

$$V(f) = \frac{K_0}{K_v} H(f) \phi_e(f)$$

$$V(f) = \frac{Jf}{K_v} L(f) \phi_e(f) \quad \text{---(12)} \quad L(f) = \frac{K_0 H(f)}{Jf}$$

From equations (10) & (12)
$$V(f) = \frac{Jf}{K_v} L(f) \frac{\phi_1(f)}{1 + L(f)}$$

If $L(f) \gg 1$, the above equation becomes

$$V(f) = \frac{Jf}{K_v} \phi_1(f)$$

$$V(f) = \frac{J2\pi f}{2\pi K_v} \phi_1(f)$$

$$V(f) = \frac{Jw}{2\pi K_v} \phi_1(f)$$

$$V(t) = \frac{1}{2\pi K_v} \frac{d\phi_1(t)}{dt} \quad \text{---(13)}$$

FM Demodulators

$\phi_1(t)$ is related to modulating wave $m(t)$ shown in equation(2)

$$\phi_1(t) = 2\pi k_f \int_0^t m(t) dt$$

$$V(t) = \frac{2\pi k_f}{2\pi K_v} \frac{d}{dt} \int_0^t m(t) dt$$

$$V(t) = \frac{k_f}{K_v} m(t)$$

The output $V(t)$ of the phase locked loop is approximately same as the original base band signal $m(t)$ except for a scaling factor K_f/K_v .

Hence frequency demodulation is accomplished.

FM Demodulators

The process of extracting an original message signal from the modulated wave is known as **detection** or **demodulation**. The circuit, which demodulates the modulated wave is known as the **demodulator**.

Let us discuss about the FM demodulators which demodulate the FM wave. The following two methods demodulate FM wave.

- Frequency discrimination method
- Phase discrimination method

Frequency Discrimination Method:

We know that the equation of FM wave is

$$s(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int m(t) dt \right)$$

Differentiate the above equation with respect to 't'.

$$\frac{ds(t)}{dt} = -A_c \left(2\pi f_c + 2\pi k_f m(t) \right) \sin \left(2\pi f_c t + 2\pi k_f \int m(t) dt \right)$$

We can write, $-\sin \theta$ as $\sin(\theta - 180^\circ)$

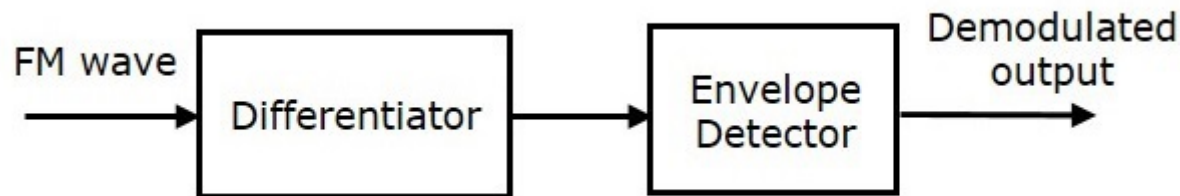
$$\frac{ds(t)}{dt} = A_c \left(2\pi f_c + 2\pi k_f m(t) \right) \sin \left(2\pi f_c t + 2\pi k_f \int m(t) dt - 180^\circ \right)$$

$$\frac{ds(t)}{dt} = A_c 2\pi f_c \left(1 + 2\pi \frac{k_f}{f_c} m(t) \right) \sin \left(2\pi f_c t + 2\pi k_f \int m(t) dt - 180^\circ \right)$$

In the last equation, the amplitude term resembles the envelope of AM wave and the angle term resembles the angle of FM wave. Here, our requirement is the modulated signal $m(t)$. Hence, we can recover it from the envelope of AM wave.

FM Demodulators

The following figure shows the block diagram of FM demodulator using frequency discrimination method.



This block diagram consists of the differentiator and the envelope detector. Differentiator is used to convert the FM wave into a combination of AM wave and FM wave. This means, it converts frequency variations of FM wave into the corresponding voltage (amplitude) variations of AM wave. We know the operation of the envelope detector. It produces the demodulated output of AM wave, which is nothing but the modulating signal.

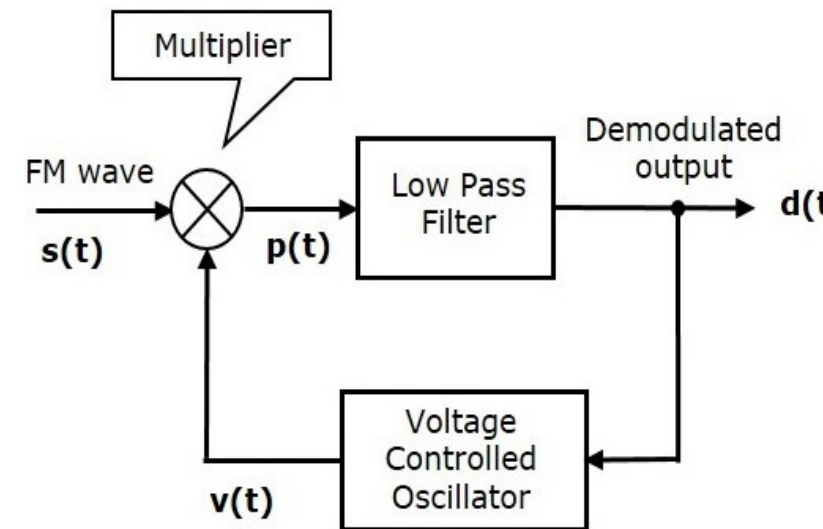
FM Demodulators

Phase Discrimination Method:

The following figure shows the block diagram of FM demodulator using phase discrimination method.

This block diagram consists of the multiplier, the low pass filter, and the Voltage Controlled Oscillator (VCO). VCO produces an output signal $v(t)$, whose frequency is proportional to the input signal voltage $d(t)$.

Initially, when the signal $d(t)$ is zero, adjust the VCO to produce an output signal $v(t)$, having a carrier frequency and -90° phase shift with respect to the carrier signal.

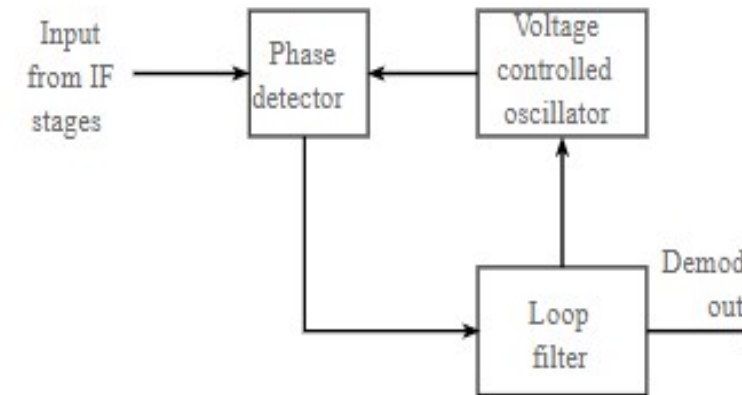


FM wave $s(t)$ and the VCO output $v(t)$ are applied as inputs of the multiplier. The multiplier produces an output, having a high frequency component and a low frequency component. Low pass filter eliminates the high frequency component and produces only the low frequency component as its output.

This low frequency component contains only the term-related phase difference. Hence, we get the modulating signal $m(t)$ from this output of the low pass filter.

PLL FM Demodulators

The phase locked loop, PLL is a very useful RF building block. The PLL uses the concept of minimising the difference in phase between two signals: a reference signal and a local oscillator to replicate the reference signal frequency. Using this concept it is possible to use PLLs for many applications from frequency synthesizers to FM demodulators, and signal reconstitution.



PLL Phase locked Loop FM demodulator

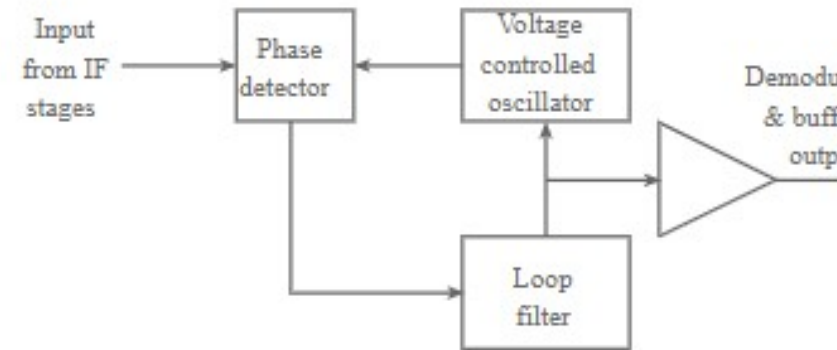
To look at the operation of the PLL FM demodulator take the condition where no modulation is applied and the carrier is in the centre position of the pass-band the voltage on the tune line to the VCO is set to the mid position. However if the carrier deviates in frequency, the loop will try to keep the loop in lock. For this to happen the frequency must follow the incoming signal, and in turn for this to occur the tune line voltage must vary.

Monitoring the tune line shows that the variations in voltage correspond to the modulation applied to the signal. By amplifying the variations in voltage on the tune line it is possible to generate the demodulated signal.

Although no basic changes to the phase locked loop are required for it to be able to demodulate FM, a buffer amplifier is typically provided from the tune line to prevent the tune line being loaded by other sections of the receiver. It provides a lower output impedance and as a result, this prevents loading from the audio amplifier from upsetting the loop in any way.

FM Demodulators

There are many different ICs that enable FM to be demodulated. One of the most popular has been the 565 that has been around for many years in a variety of forms. Even though the circuit is quite old, it performs well, and often little will be gained by going to other chips.



PLL Phase locked Loop FM demodulator with buffered output

Pulse Modulation

Multiplexing : Transmits multiple (many) signals over a single channel.

Need of Multiplexing :-

- Transmitting two or more signals simultaneously can be accomplished by setting up one transmitter-receiver pair for each channel, but this is an expensive approach.
- A single cable or radio link can handle multiple signals simultaneously using a technique known as **multiplexing**.
- Multiplexing permits hundreds or even thousands of signals to be combined and transmitted over a single medium.
- Cost savings can be gained by using a single channel to send multiple information signals.

The Concept of Multiplexing

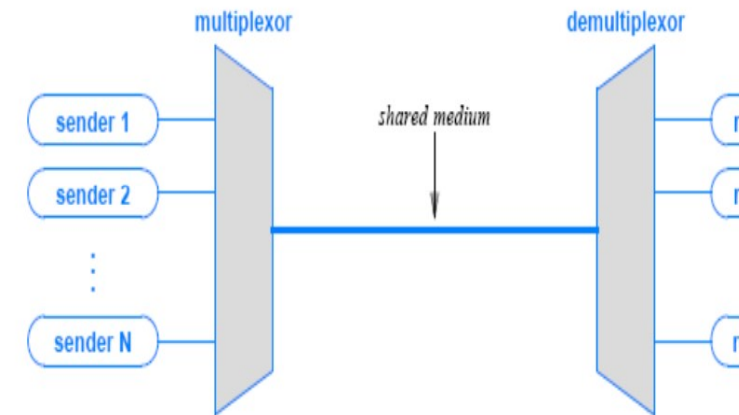


Figure 11.1 The concept of multiplexing in which independent pairs senders and receivers share a transmission medium.



Pulse Modulation

The Basic Types of Multiplexing

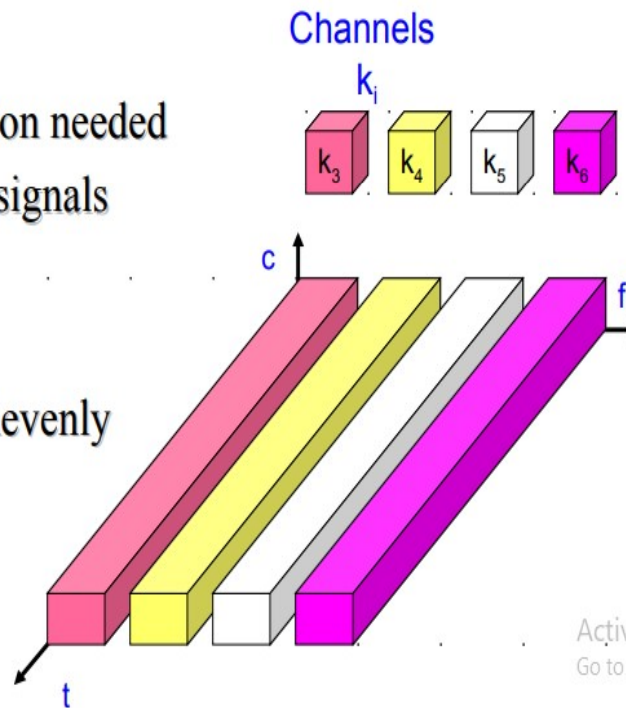
There are four basic approaches to multiplexing that each have a set of variations and implementations

- Frequency Division Multiplexing (FDM)
- Wavelength Division Multiplexing (WDM)
- Time Division Multiplexing (TDM)
- Code Division Multiplexing (CDM)
- TDM and FDM are widely used
- WDM is a form of FDM used for optical fiber
- CDM is a mathematical approach used in **cell phone** mechanisms

Pulse Modulation

Frequency Division Multiplex

- Separation of spectrum into smaller frequency bands
- Channel gets band of the spectrum for the whole time
- Advantages:
 - no dynamic coordination needed
 - works also for analog signals
- Disadvantages:
 - waste of bandwidth if traffic distributed unevenly
 - inflexible
 - guard spaces



Frequency Division Multiplexing (FDM)

- Each signal is allocated a different frequency band
- Usually used with analog signals
- Modulation equipment is needed to move each signal to the required frequency band (channel)
- Multiple carriers are used, each is called sub-carrier
- Multiplexing equipment is needed to combine the modulated signals

Pulse Modulation

Time division multiplexing (TDM):

In TDM, multiplexing is based on time. The sampled PAM (pulse means on and off) waveform is off for most of the time. During the off period, the channel can be used to transmit samples of other waveforms. The concept of interleaving samples from several signals into a single waveform is called TDM.

Time Division Multiplexing

Definition: Time Division Multiplexing (TDM) is the time interleaving of samples from several sources so that the information from these sources can be transmitted serially over a single communication channel.

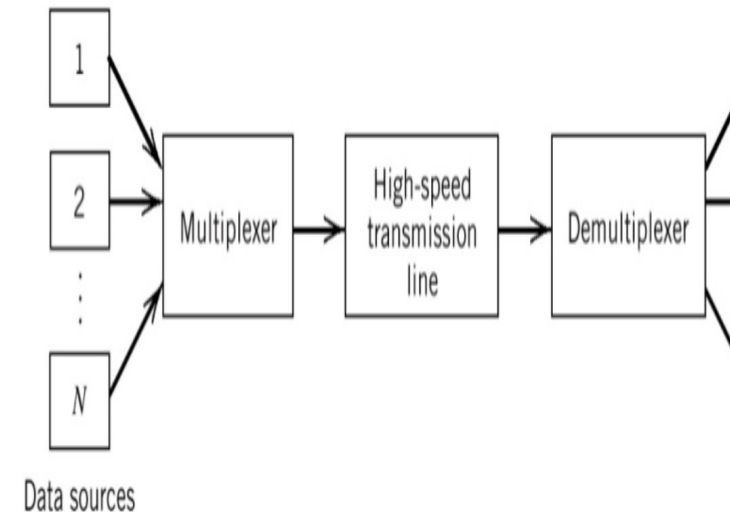
At the Transmitter

- Simultaneous transmission of several signals on a time-sharing basis.
- Each signal occupies its own distinct time slot, using all frequencies, for the duration of the transmission.
- Slots may be permanently assigned on demand.

At the Receiver

- Decommulator (sampler) has to be synchronized with the incoming waveform → *Frame Synchronization*
- Low pass filter
- ISI – poor channel filtering
- Feedthrough of one channel's signal into another channel -- *Crosstalk*

Time Division Multiplexing



Applications of TDM: Digital Telephony, Data communications, Satellite Access, Cellular radio.

Activ
Go to 5

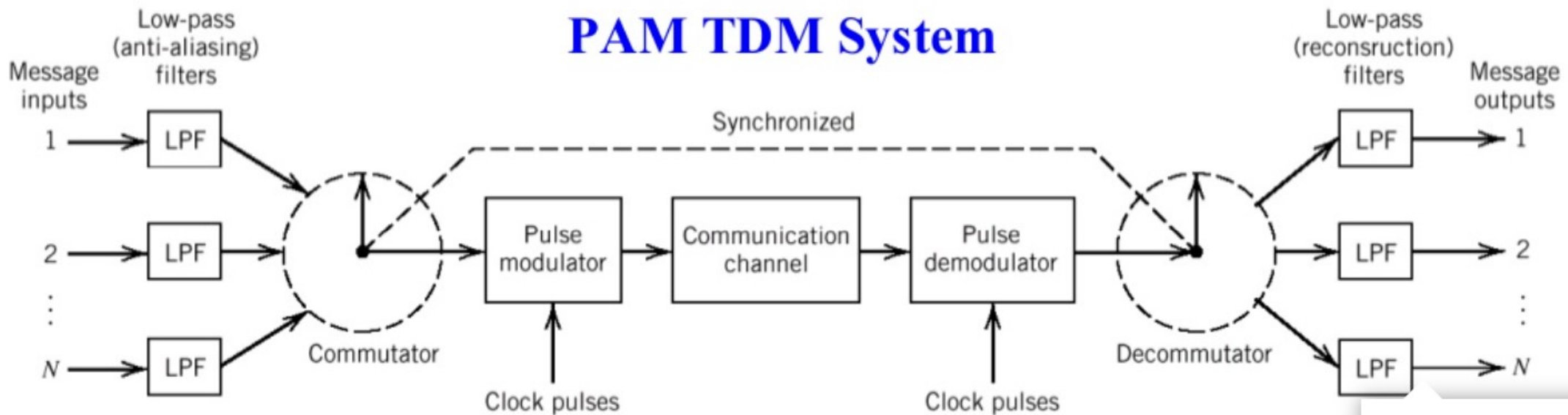


Pulse Modulation

Block diagram of TDM system

Clip slide

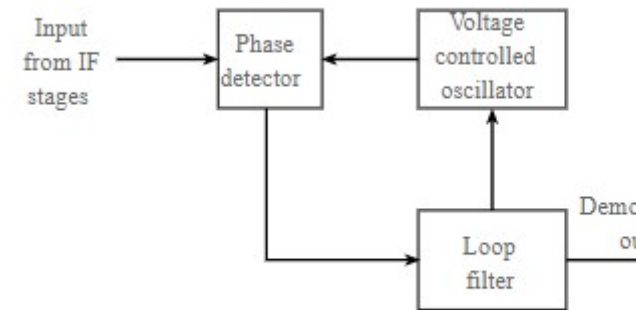
PAM TDM System



FM Demodulators

PLL FM Demodulators

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